Weather Forming Ultra Low Frequency Electromagnetic Waves at Interaction with Local Inhomogeneous Winds in the Ionosphere

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Abstract. The generation and further dynamics of planetary ULF waves are investigated in the rotating dissipative ionosphere in the presence of a smooth inhomogeneous zonal wind (shear flow). Planetary ULF waves appear as a result of the interaction of the medium with the spatially inhomogeneous geomagnetic field. An effective linear mechanism responsible for the intensification and mutual transformation of large scale magnetized Rossby type and small scale inertial waves is found. For shear flows, the operators of the linear problem are not self-adjoint, and therefore the eigenfunctions of the problem maybe non-orthogonal and can hardly be studied by the canonical modal approach. Hence it becomes necessary to use the so-called nonmodal mathematical analysis. The nonmodal approach shows that the transformation of wave disturbances in shear flows is due to the non-orthogonality of eigenfunctions of the problem in the conditions of linear dynamics. Using numerical modeling, it is illustrated the peculiar features of the interaction of waves with the background flow as well as the mutual transformation of wave disturbances in the ionosphere.

Keywords: ULF electromagnetic wave, Shear flow, non-modal approach, Nonlinear solitary vortex structures.

Introduction

For studying the dynamics of large-scale planetary processes in the ionosphere it is necessary to take into consideration the inhomogeneous and non-stationary properties of a wind process, a turbulent state of the lower ionosphere and the influence of inhomogeneous electromagnetic forces. These factors, which are especially strongly pronounced because of a low density of the medium in the ionosphere and a relatively high conductivity of the ionospheric gas, may cause essential deflections of the real wind (usual Rossby planetary wave) from geostrophic motion. Hence the general ionospheric circulation has certain peculiarities that are not observed in the conditions of the troposphere.

The action of a geomagnetic field leads, on the one hand, to the inductive damping of Rossby type planetary waves, which is connected with Pedersen or transverse (relative to a magnetic field) conductivity, and, on the other hand, to the gyroscopic effect caused by the Hall conductivity of the ionosphere and having an impact on disturbances like the Coriolis force. As a result of the joint action of the spatially inhomogeneous Coriolis force and the electrodynamic (connected with the geomagnetic field) force,. in the ionosphere there may exist a new type of waves which physically differ from the usual Rossby wave and which are called magnetized Rossby or Rossby type waves.

Observations (Gossard and Hooke, 1975; Pedlosky 1978) show that the atmospheric and ionospheric layers always have spatially inhomogeneous zonal winds (shear flows) produced by a nonuniform heating of the atmospheric layers by solar radiation. In this connection, it becomes important to investigate the problem on generation and evolution of usual and magnetized Rossby waves at their interaction with the inhomogeneous zonal wind (shear flow).

The canonical (modal) investigation of linear wave processes (spectral expansion disturbances with respect to time followed by analysis of the eigenvalues) in shear flows does not take into account a highly important physical process, namely, the mutual transformation of wave modes (Graik, Criminale 1986; Trefethen et al. 1993).

A strict mathematical description of the peculiarities of shear flows revealed (Trefethen, et al. 1993) that in the case of canonical (modal) analysis of linear processes the operators figuring in dynamic equations are not self-conjugate (Graik, Criminale, 1986) and, as a result, the eigenfunctions of the problem are not orthogonal to each other – they strongly interfere with each other. Thus, for a correct description of phenomena it becomes necessary to carry out accurate calculations of effects of the interference of eigenfunctions, which sometimes turns out to be the problem of insurmountable difficulty.

There also exists another so-called nonmodal analysis of linear processes in shear flows. With this approach, the modified initial problem (Cauchy problem) is solved by tracing the evolution of spatial Fourier-harmonics (SFH) of wave disturbances in time and not using any spectral expansion with respect to time (Reddy, Schmid, Hennigson, 1993; Chagelishvili, Rogava, Tsiklauri 1996). Being an optimal "language", the nonmodal approach much simplifies a mathematical description of the dynamics of shear flow disturbances and makes it possible to reveal the key phenomena (caused by the nonorthogonality of linear dynamics) which have escaped the notice in the case of modal analysis.

In this paper we investigate the linear evolution of Rossby type waves in shear zonal flows (winds) in D, E, and F-regions of the ionosphere. In dynamic equations, the disturbed magnetohydrodynamic values are represented through SFH. This corresponds to nonmodal analysis in the coordinate system which moves with the background wind. This spatial Fourier expansion allows us to replace, in the basic equations, the spatial inhomogeneity connected with the inhomogeneity of the basic zonal flow by the time-dependent inhomogeneity and to trace how the SFH of disturbances evolved in time.

Initial equations and the basic principles of nonmodal analysis

In this paper we are interested mainly in large-scale (planetary) wave motions in the ionospheric medium (consisting of electrons, ions and neutral particles), which have a horizontal linear scale L_h of order 10^3 km and higher, a vertical scale L_v of altitude scale order H_0 ($L_v \approx H_0$) and a time scale τ of half-day order and higher. It is such motions that are connected with global distributions of the ionospheric structure and its large-scale daily, seasonal, 27-day and other variations. According to experimental data (Gossard and Hooke, 1975; Pedlosky 1978) in ionospheric large-scale motions the relation of the characteristic vertical velocity V_v to the horizontal one V_h is small: $V_v / V_h \le L_v / L_h < 10^{-2}$. The latter relation implies that large-scale motions in the ionosphere are mostly quasi-horizontal. The dynamic properties of such a medium are defined by the neutral component because of the fulfillment of the condition $N_{e,i} / N_n << 1$ (where N_e, N_i, N_n are the concentration of electrons, ions and the neutral component, respectively). The presence of charged particles makes the considered medium electroconductive.

In the light of the above reasoning, the basic properties of a Rossby type planetary wave in the ionosphere better to considere the equation for the horizontal velocity $\mathbf{V}_\perp(V_x, V_y)$ as initial one, where acceleration is defined by the pressure gradient, Coriolis force, volumetric electrodynamic force and viscous friction (Dokuchaev, 1959; Gossard and Hooke, 1975; Pedlosky 1978).

The geomagnetic field $\mathbf{B}_0(B_{0x}, B_{0y}, B_{0z})$ is dipolar and in the chosen coordinate system has the following components (Dokuchaev, 1959):

$B_{0x} = 0,\ B_{0y} = -B_e \sin\theta'$, $B_{0z} = -2B_e \cos\theta'$,

where $B_e \approx 3{,}5 \times 10^{-5}$ Tesla (T) is the value of geomagnetic field induction at the equator. In this case, the total induction of the geomagnetic field is $B_0 = B_e\left(1 + 3\cos^2\theta'\right)^{1/2}$, while $\theta' = \pi/2 - \varphi',\ \varphi'$ is the geomagnetic latitude. In the same coordinate system, the components of the angular velocity vector of the Earth's rotation $\mathbf{\Omega}_0\left(\Omega_{ox}, \Omega_{oy}, \Omega_{oz}\right)$ can be written as

$$\Omega_{0x} = 0, \quad \Omega_{0y} = \Omega_0 \sin\theta, \quad \Omega_{0z} = \Omega_0 \cos\theta .$$

It is assumed that the geographical and geomagnetic latitudes φ and φ' coincide $(\varphi = \varphi', \theta = \theta')$ and disturbances occur in the neighborhood of the latitude $\varphi_0 = \pi/2 - \theta_0$. Further, system (2.1)–(2.5) is linearized against the background of a plane zonal shear flow (wind) $\mathbf{V}_0$: for hydrodymic velocity $\mathbf{V} = \mathbf{V}_0 + \mathbf{V}'(x, y)$, for medium density $\rho = \rho_0 + \rho'(x, y)$ and for preasure $P = P_0 + P'(x, y)$, where the values with a prime are the disturbed ones, while the mean (background) values have the sub-index zero (for simplicity, in the sequel we omit the prime of the perturbed values). Thus the initial system of equations for large-scale small (linear) disturbances can be written in the form

$$\frac{d\mathbf{V}_\perp}{dt} + (\mathbf{V}_\perp \nabla)\mathbf{V}_0 = -\frac{\nabla P}{\rho_0} - 2[\mathbf{\Omega} \times \mathbf{V}_\perp] - \frac{\sigma_\perp}{\rho_0 c^2}\left(B_0^2 \mathbf{V}_\perp - B_{0y} V_y \mathbf{B}_0\right) + \\ + \frac{B_0 \sigma_H}{\rho_0 c^2}[\mathbf{V} \times \mathbf{B}_0] + \nu \Delta \mathbf{V}_\perp , \qquad (1)$$

$$\gamma \frac{d\rho}{dt} + \gamma(\mathbf{V}_\perp \nabla)\rho_0 + \rho_0 \mathrm{div}\mathbf{V}_\perp = 0, \qquad (2)$$

$$\frac{dP}{dt} + (\mathbf{V}_\perp \nabla)P_0 + P_0 \mathrm{div}\mathbf{V}_\perp = 0. \qquad (3)$$

To proceed with our analysis of the peculiar properties of a magnetized Rossby wave in the ionosphere, it is convenient to introduce the coordinate system with the moving axes $X_1 O_1 Y_1$, whose origin O_1 and Y_1-axis coincide with their counterparts of the equilibrium local system XOY , while the X_1-axis moves together with the undisturbed (background) flow. For our problem, this is equivalent to the replacement of the variables $x_1 = x - ayt, \quad y_1 = y, \quad t_1 = t,$ or to

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_1} - ay\frac{\partial}{\partial x_1} , \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial x_1} , \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial y_1} - at_1\frac{\partial}{\partial x_1} .$$

In the new variables, the initial equations take the form:

$$\frac{\partial V_x}{\partial t_1} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x_1} - \frac{\sigma_\perp B_0^2}{\rho_0 c^2}V_x + \left(2\Omega_{0z} + \frac{\sigma_H B_0 B_{0z}}{\rho_0 c^2} - a\right)V_y + \\ + \nu\left\{\frac{\partial^2}{\partial x^2} + \left(\frac{\partial}{\partial y_1} - at_1\frac{\partial}{\partial x_1}\right)^2\right\}V_x , \qquad (4)$$

$$\frac{\partial V_y}{\partial t_1} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x_1} - \frac{\sigma_\perp B_{0z}^2}{\rho_0 c^2}V_y - \left(2\Omega_{0z} + \frac{\sigma_H B_0 B_{0z}}{\rho_0 c^2}\right)V_x \\ + \nu\left\{\frac{\partial^2}{\partial x^2} + \left(\frac{\partial}{\partial y_1} - at_1\frac{\partial}{\partial x_1}\right)^2\right\}V_y , \qquad (5)$$

$$\frac{\partial P}{\partial t_1} + P_0\left\{\frac{\partial V_x}{\partial x_1} + \left(\frac{\partial}{\partial y_1} - at_1\frac{\partial}{\partial x_1}\right)V_y\right\} = 0 . \qquad (6)$$

The above replacement of the variables does not mean that we have physically passed over to a new counting system, since the values V_x, V_y, P in the initial equations are equivalent to the velocity and pressure components of wave disturbance in the Cartesian system XOY. The coefficients of the initial system of linear equations depended on the spatial coordinate

y. The above mathematical transformations have changed this spatial inhomogeneity for time inhomogeneity. Thus the coefficients of system have become independent of the spatial variables x_1, y_1 and we are able now perform Frourier analysis of these equations with respect to the spatial variables (x_1, y_1), and consider the time evolution of these SFH separately:

$$\begin{Bmatrix} V_x(x_1, y_1, t_1) \\ V_y(x_1, y_1, t_1) \\ P(x_1, y_1, t_1) \end{Bmatrix} = \int_{-\infty}^{\infty}\int\int dk_{x1} dk_{y1} \begin{Bmatrix} \tilde{V}_x(k_{x1}, k_{y1}, t_1) \\ \tilde{V}_y(k_{x1}, k_{y1}, t_1) \\ \tilde{P}(k_{x1}, k_{y1}, t_1) \end{Bmatrix} \times \exp(ik_{x1}x_1 + ik_{y1}y_1).$$

From initial equation (4)-(6), substituting the notations $\Omega = \mathrm{rot}_z \mathbf{V}_\perp$ and $\xi = \mathrm{div}\mathbf{V}_\perp$ and passing to dimensionless variables and parameters, after some transformation for each SFH we will have:

$$\frac{\partial\Omega}{\partial\tau} = \left[i\frac{k_x}{k^2(\tau)}(\beta + \beta_{Hz}) - b_{\perp z} - \frac{k_y^2(\tau)}{k^2(\tau)} b_{\perp y} - \nu k^2(\tau) \right]\Omega - \left[1 - S - i\frac{k_y(\tau)}{k^2(\tau)}(\beta + \beta_{Hz}) + b_{Hz} - \frac{k_x k_y(\tau)}{k^2(\tau)} b_{\perp y} \right]\xi, \quad (7)$$

$$\frac{\partial\xi}{\partial\tau} = -\left[2S\frac{k_x k_y(\tau)}{k^2(\tau)} - i\frac{k_x}{k^2(\tau)}(\beta + \beta_{Hz}) - i\frac{k_y(\tau)}{k^2(\tau)}\beta_{\perp z} + b_{\perp z} + \frac{k_x^2}{k^2(\tau)} b_{\perp y} + \nu k^2(\tau) \right]\xi + \left[1 - 2S\frac{k_x^2}{k^2(\tau)} - i\frac{k_y(\tau)}{k^2(\tau)}(\beta + \beta_{Hz}) + \frac{k_x}{k^2(\tau)}\beta_{\perp z} + b_{Hz} + \frac{k_x k_y(\tau)}{k^2(\tau)} b_{\perp y} \right]\Omega + k^2(\tau)P, \quad (8)$$

$$\frac{\partial P}{\partial \tau} = -\delta\xi. \quad (9)$$

In the space of wave numbers, the density of total energy of wave disturbances, have the form $E[k] = \frac{\Omega\Omega^*}{k^2(\tau)} + \frac{\xi\xi^*}{k^2(\tau)} + \frac{PP^*}{\delta}$, where the asterisk denotes the complex conjugacy.

Thus, the density of total energy of wave disturbance consists of three parts, where the first term is the energy of the vortical part of disturbances, $E_v = \Omega\Omega^*/k^2(\tau)$; the second term is the compressible part of the energy, $E_c = \xi\xi^*/k^2(\tau)$; the third term is the elastic (potential) energy (due to disturbance elasticity), $E_e = PP^*/\delta$. In the absence of a shear flow $(S = 0)$ and dissipative processes $(\nu = 0, \sigma_\perp = 0)$, the total energy density of the considered wave disturbances in the ionosphere preserves its value $\partial E(\tau)/\partial\tau = 0$.

We carried out the numerical solutions and experiments of (7)-(9) equations for different values of the medium and wave parameters the following features were revealed.

Analysis of numerical experiments

In this paper we want to discuss the mistake made in describing the evolution of Rossby type waves in the presence of zonal shear flows.

Intensification.

At the initial time moment, only the low-frequency planetary Rossby wave with a large value of the meridianal wave vector $k_y(0)$, $k_y(0)/k_x = 50 >> 1$ and $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$, was excited. When $k_y(0)/k_x >> 1$, the Rossby wave is mainly vortical and practically incompressible. In the incompressible stage, wave excitations may absorb the background flow energy only if $k_y(\tau) \approx k_x$. Indeed, due to the linear drift, $k_y(\tau)$ decrease with a lapse of time, but for $k_y(\tau) >> k_x$ the energy exchange between the background flow and the SFH of the Rossby wave is not essential. For times when already $k_y(\tau) \approx k_x$ the SFH of the Rossby wave actively absorbs the shear energy and gains in intensity, i.e. the SFH is now in the region of intensification. The SFH intensification stops at the time moment $k_y(\tau^*) = 0$. Then, for $k_y(\tau)/k_x < 0$ or in a time interval $\tau^* < \tau \le \tau_1$, it begins to give back some part of the energy to the mean flow (see figure 1).

Transformation.

With the evolution of the initial excitement, the share of the vortex component in the total energy keeps decreasing until it becomes negligibly small (for $\tau_2 \sim 80$) and a greater part of the Rossby wave energy is pumped into the energy of inertial waves. Thus the Rossby wave transforms to inertial waves. The total energy and the SFH now have high-frequency oscillations. Thus, if for $\tau = 0$ the energy is concentrated in vortical low-frequency modes (Rossby waves), for $\tau >> \tau^*$ the whole energy is concentrated in potential high-frequency disturbances, i.e. in inertial waves. Transformation of Rossby type waves to inertial ones starts from the moment $\tau = \tau^*$ and goes on within a limited time interval in which the conditions of transformation are fulfilled and these two branches get interconnected. A greater part of the Rossby wave energy undergoes transformation. It can be said that by the time moment $\tau = \tau_1$ only the (inertial) wave remain in the flow. With a lapse of time, the latter wave intensifies by absorbing the shear energy.

Damping of large-scale wave disturbances in the shear flow

As has already been mentioned, in the shear flow we observe the SFH drift in the space of wave numbers. Thus, with a lapse of time, the radial component of the SFH wave vector $k_y(\tau) = k_y(0) - Sk_x\tau$ increases, i.e. the disturbance length decreases along the meridian (for $\tau \to \infty$, $\ell_y = 2\pi/|k_y(\tau)| \to 0$). Usually, in a solid medium the subdivision of scales takes place at the expense of nonlinear processes. However in our case a monotone decrease of disturbance scales occurs in the linear regime. For short-wave disturbances, the influence of dissipative processes (viscosity in our case) is essential. Due to dissipation, the disturbance energy is transferred in the form of heat to the medium and, eventually, a practically complete damping of wave disturbances takes place.

Therefore the pumping of shear flow energy to the wave perturbation energy and the mutual transformation of modes followed by their dissipation in the medium are permanent processes, which may to a strong heating of the medium. It is obvious that the heating intensity depends on the initial disturbance level and the shear flow parameter S.

Conclusion

In this paper we investigate the linear stage of the evolution of SFH of a magnetized Rossby wave and inertial wave disturbances in the dissipative ionosphere in the presence of a shear flow (smooth-inhomogeneous zonal wind). Based on the numerical solution and theoretical analysis of the corresponding system of dynamic equations, new mechanisms are found, which account for the pumping of shear flow energy to wave disturbance energy, an extremal intensification (by several orders) of waves, the mutual transformation of eigenmodes and the conversion of perturbation energy to heat.

The intensification of a magnetized Rossby wave and an inertial wave may take place for certain values of the parameters of the medium, shear and waves. This makes an unusual way of shear flow heating in the ionosphere: waves draw up the shear flow energy and pump it through the mutual linear transformation and linear drift of SFH in the space of wave numbers (subdivision of disturbance scales) to the damping domain. Finally, the viscosity and inductive damping convert this pumped energy to heat. The process is permanent and may lead to a strong heating of the medium. The heating intensity depends on the initial disturbance level and shear flow parameters.

A remarkable feature of a shear flow is the diminution of wave disturbance scales in the linear stage, which is due to a linear drift of disturbance SFH in the space of wave numbers and, accordingly, to the pumping of energy to the dissipation region (with short scales).

The intensification of wave disturbance SFH and the mutual transformation of modes take place within a limited time interval (transiently) as long as the corresponding conditions of intensification and a sufficiently strong interconnection of modes are fulfilled.

The mutual transformation of eigenmodes (of Rossby and inertial waves) may take place even in the spatial-homogeneous ionosphere ($\rho_0 = \mathrm{const}$), when the background wind velocity is inhomogeneous. We should emphasize the fact that this transformation mechanism was revealed in the framework of nonmodal mathematical analysis (these processes were not taken into account in the case of a more traditional modal approach).

The character of the wave transformation mechanism considered in this paper is essentially different from the previously known linear mechanism of wave transformation in the inhomogeneous plasma (Erokhin, Moiseev, 1973). The transformation of waves in the case of medium density inhomogeneities takes place in a limited space (across the density inhomogeneity) as long as this inhomogeneity exists, while in our case the transformation of linear waves occurs throughout the shear flow volume but in a limited time interval (transiently). It is obvious that for this phenomenon to take place it is necessary that at least two wave modes would exist in the medium.

The effect of the revealed mutual transformation of Rossby type waves and inertial waves in the ionosphere with an inhomogeneous zonal wind makes us revise some notions existing in dynamic meteorology and in the models of general circulation of the atmosphere, ocean, ionosphere and magnetosphere with the participation of Rossby type planetary waves. This especially concerns the interpretation of experimental and observation data, when it is necessary to take into account a possibility of mutual transformation of waves with different time and spatial scales in shear flows.

The presence of the electromagnetic pondermotive force, i.e. of an inhomogeneous geomagnetic field, Hall and Pedersen currents in different ionospheric layers increases the effectiveness of interaction and energy exchange between wave disturbances and the background shear flow.

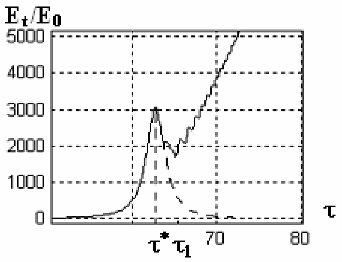

Fig. The time dependence of the total SFH energy

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REFERENCES

Chagelishvili, G. D., Rogava A. D Tsiklauri M.. G.: 1996, Phys. Rev. E, 53, pp. 6028 – 6031.

Dokuchaev V.P,: 1959 Bull. AS of SSSR. Ser. Geophys.. 5, 783-787.

Erokhin, N.S., Moiseev S.S.:1973 Questions of the plasma theory, Atomizdat, 7, 146—204.

Gossard E. and Hooke, W. 1975 "Waves in the Atmosphere", Elsevier, Amsterdam,

Graik, A. D., Criminale W. O,: 1986 Proc. R. Soc. London, Ser. A, vol. 406, 13 – 21.

Pedlosky, J.: 1978 Geophysical Fluid Dynamics. Springer-Verlag, New York.

Reddy, S. C., Schmid P. J., Hennigson D. S,:1993 SIAM. J. Appl. Math., vol. 53. 15 – 23.

Trefethen, L. N., Trefethen, A. E., Reddy S. C., Driscoll T. A.: 1993 Science, 261, 578 – 584.