Abstract. Presently there are two schools of thought viz., turbulent dynamo and MHD oscillation mechanisms that explain the solar cycle and activity phenomena. Both the mechanisms are critically examined and fundamental difficulties are presented. By keeping in mind the more advantages of having MHD oscillation mechanism, compared to the turbulent dynamo mechanism, the following new ideas on the genesis of the solar cycle and activity phenomena are presented. The inevitability of the most likely existence of a combined poloidal and toroidal magnetic field structure in the solar interior is proposed. Owing to the suitable poloidal part of the steady field structure, the Alfvén wave perturbations of long periods (~22 years) that excite in the solar core travel first to the poles in both the hemispheres and later reach the equator. While traveling towards the surface, the Alfvén wave perturbations along the weak poloidal field structure in turn perturb the embedded strong toroidal field structure producing sunspots, especially in the convective envelope, that travel to the surface due to buoyancy along isorotational contours. With a realistic density structure of the solar interior, the computation of Alfvén wave travel times along different field lines of the poloidal field structure [1] yields almost similar periods (~22 years) explaining the constancy of 22 years periodicity of the odd degree modes obtained from the Spherical Harmonic Fourier analysis of the surface magnetic field. The observed quasi-periodicities of solar activity indices in the range of 1-5 years are explained as due to the Alfvén wave perturbation of the strong toroidal field structure. The variation of the long period solar cycle and activity phenomena such as the Maunder and the grand minima is explained to be due to the coupling of long period poloidal and toroidal MHD oscillations.

Keywords: The Sun, solar cycle, solar dynamo, solar MHD oscillations, Maunder minimum

Introduction

Since the discovery of sunspots by Galileo, the physics of solar cycle and activity phenomena is not understood completely and remains one of the major unsolved problems in solar physics. With the recent overwhelming evidences that the solar cycle and the activity phenomena strongly influence the earth’s environment and climate [2], it is necessary to understand the physics involved.

Presently there are two schools of thought - the turbulent dynamo and the MHD oscillatory mechanisms - on the genesis of the solar cycle and activity phenomena. Although the turbulent dynamo models explain qualitatively many of the observed solar cycle and activity phenomena, there are several difficulties and limitations [3-8] in their application to the solar cycle. Details of the theory of turbulent dynamo mechanism and its limitations for application to physics of the solar cycle and activity phenomena can be found in [9]. In this study, I mainly concentrate on the MHD oscillatory theory and show with new ideas that many of the observed solar cycle and activity phenomena can be explained.

In section 2, I briefly explain the theory of MHD oscillations. In section 3, the seminal work of Alfvén on the theory of solar cycle is revisited. In section 4, new views on the genesis of the solar cycle and activity phenomena are proposed and important observations of solar cycle and activity phenomena are explained. Conclusions of this study are given in section 5.

Theory of MHD Oscillations

In the electrically conducting magnetized plasma, there are three kinds of MHD waves, viz., (i) Alfvén wave, (ii) slow MHD wave and, (iii) fast MHD wave. Since the sun is a dynamic body such disturbances in the medium always exist, which perturb the magnetic field structure leading to the generation of Alfvén waves. Alfvén waves are of two types [10], viz., shear Alfvén waves due to incompressibility and compressible Alfvén waves due to compressibility. The shear Alfvén waves are transverse waves that travel along the field lines, whereas the compressible Alfvén waves consist of both longitudinal and transverse waves. Since the time scales of compressible waves (~ 5 min) due to density perturbations are very much smaller than the solar cycle time scales (~ 22 years), the condition of incompressibility applies and shear Alfvén waves are best suited for the present study.

Observed periodic behavior of the large-scale magnetic field of the sun is viewed as a consequence of MHD oscillations in the presence of a large-scale steady (diffusion time scale ~ billion years) magnetic field. These theories recognize the fact that most of the observed fields at the surface (including those in the polar regions) are in the form of bipolar regions. The MHD oscillations must be azimuthal perturbations of the ambient steady poloidal magnetic field structure. Amplification of the toroidal field can result from the azimuthal perturbations of the ambient steady poloidal magnetic field. Any such perturbations of the field lines would eventually lead to
MHD waves. The waves travel along the field lines of the steady poloidal field structure and are reflected due to density gradients near the surface. Superposition of many such traveling waves leads to stationary or standing oscillations. The strong fields needed for the activity result from the constructive interference of these waves.

For axisymmetric magnetic field structure and in cylindrical geometry, the MHD wave equation [11] is given by

$$\frac{\partial^2 \Omega}{\partial t^2} = \frac{B_p}{4 \pi \rho} \frac{\partial^2 \Omega}{\partial s^2}$$

(1)

Where $\Omega$ is angular velocity, $B_p$ is poloidal component of the steady magnetic field structure and $\rho$ is the density of the ambient plasma. In addition we have a similar equation by replacing $\Omega$ by toroidal field $B_p$. These two equations imply that the changes in either $\Omega$ or $B_p$ propagate with the local Alfvén speed $V_A = B_p / \sqrt{4 \pi \rho}$ determined by the steady poloidal field structure. Since the perturbations are in the azimuthal direction, such a wave equation is called torsional MHD wave equation. In fact, in the following subsection, we use this equation for checking the admissibility of global torsional MHD oscillations in various models of the steady magnetic field structures in the solar interior. Alfvén [12] and Walen [13] were the pioneers to propose this theory and latter their ideas were revived by many authors [14-18, 4, 1, here onwards HG95].

Alfvén’s Theory of Solar Cycle

In his seminal work, Alfvén [12] assumed that: (i) the sun consists of large-scale dipole magnetic field structure in the interior whose magnetic axis coincides with the rotation axis, (ii) a magnetic disturbance somewhere in the deep interior travels with Alfvén speed $V_A$ along the field lines and reach the surface, (iii) excitation of MHD waves is due to the turbulence that is created by the differences in the velocity gradients of the isorotational contours and, (iv) coupling between the neighboring field lines expected to transfer the oscillations towards all parts of the sun.

For a polytropic density variation, and for the dipole magnetic field structure with a dipole moment $4.2 \times 10^{33}$ G cm$^3$, Alfvén computed the travel times along different field lines and found that ~70 years for the field lines near the pole, and ~80 years for the field lines near the equator. Since these periods did not agree with the 22 year period, he concluded that the 22 year period must be the resonance period of some lines of force in the interior. In addition, Alfvén’s theory also explained the observed propagation of sunspot zones and opposite polarities of the sunspots. Alfvén computed the dependence of the sunspot frequency with respect to latitude and found almost similar results as that of the observation. By the theory of standing oscillations along different field lines, Alfvén explained the observed fact that during a particular cycle the sunspots in both the hemispheres have opposite polarities. Assuming that the perturbations in the interior are irregular, he made an attempt to explain the long period sunspot activity. Though Alfvén’s theory appears to explain most of the observations of the solar cycle and activity phenomena, there are some serious and fundamental difficulties [9].

New Ideas on the Physics of the Solar Cycle

Firstly, we have to admit that MHD oscillatory theories have the following three main difficulties: (i) the lack of observational evidence of magnetic field structure of primordial origin, (ii) difficulty in believing that such a perturbed poloidal field structure of weak general magnetic field ($\sim 1$ G) can produce sunspot activity of strong magnetic field ($\sim 103$ G), (iii) owing to the strong dissipation in the convective envelope, long period ($\sim 22$ years) MHD oscillations cannot be maintained for the next cycle.

Existence of Combined Poloidal and Toroidal Magnetic Field Structure in the Solar Interior

The likely existence of a large-scale poloidal magnetic field structure can be confirmed from the white light pictures (see Fig. 1 of [19-20]; see the Figs. 3 and 8 of [21]) during total solar eclipse around solar minimum. Though direct measurements of such a large-scale weak magnetic field ($\sim 1$ G) are lacking, indirectly, from the helioseismic rotational isocontours we ([4]; HG95) proposed a most likely poloidal magnetic field structure of primordial origin in the solar interior.

Observations show that the poloidal field is very weak ($\sim 1$ G) compared to the strength of rotation, hence the poloidal field must isorotate with the internal rotation of the plasma. This implies that the geometrical poloidal field structure must be similar to the geometrical structure of the internal isorotational contours as inferred from helioseismology. In fact it is true for the rotational isocontours (as inferred from the helioseismology) in the convective envelope where the inferred rotational isocontours are reliable. In the previous study and by using Chandrasekhar’s MHD equations, we ([4]; HG95) modeled steady part of the poloidal field structure and found the diffusion time scales to be ~ billion years. Gough and McIntyre [22] also have proposed the inevitability of such a poloidal field structure in the radiative interior. Interesting and crucial result of HG95 model of poloidal magnetic field structure is that it asymptotically approaches a uniform field at large distances that merges with the interstellar field and the strength of such a uniform field structure is independent of the latitude. Recent Ulysses observations ([23, 24]) of large scale magnetic field structure at 1.3-5.3 AU confirms HG95 model that is a reasonable representation of existence of the poloidal field structure of primordial origin in the solar interior. With reasonable assumptions and approximations and, by using MHD equations, we [3] consistently obtained the solution for both the internal rotation and the toroidal component of the magnetic field structures in the convective envelope. The toroidal field structure in the convective envelope has a quadrupole filed like geometric structure and the field strength varies from ~ $10^3$ G near base of
The convection zone to ~ 1G near the surface. In fact recent helioseismic inferences (25) yield almost similar strength of the toroidal magnetic field structure in the convective envelope. For the sake of stability (11, 26, 27) also, such a combined poloidal and toroidal field structure is necessary in the solar interior.

Hence, the sun may be pervaded by a combination of large-scale steady poloidal and toroidal magnetic field structures (both of which may of primordial origin and diffusion time scales are ~ billion years). Hence from the theoretical investigations and from the helioseismic inferences, one can reasonably accept the existence of such a combined magnetic field structure in the solar interior. Thus, the first difficulty in MHD oscillatory theory can be removed.

**Genesis of the Solar Cycle and Activity Phenomena**

The second difficulty of the oscillatory model can be removed as follows. Following Alfvén [12], any perturbations (for example [28]) near the center travel along and perpendicular to the poloidal field structure and, the coupling between neighboring field lines transfer the perturbed energy to all parts of the sun. An interesting property of the shear Alfvén waves is that the magnetic and velocity perturbations are perpendicular to the magnetic field lines and travel along the field lines.

![Image](image-url)

**Fig.1.** Meridional cross section of steady part of poloidal component of the magnetic field structure in one quadrant of the solar interior [1]. The field line A on the y axis is parallel to the rotation axis and the field lines L and M are close to the equator. This figure is reproduced from, and all rights are reserved by, the Astrophysical Journal.

That means the Alfvén waves while traveling along the field lines perturb in turn the neighboring field lines. If one believes that the sun has a magnetic field structure similar to the one proposed by HG95, then the field lines that pass through the north and south poles (field line represented by ‘A’ in Fig.1) in both the hemispheres experience the Alfvén wave perturbations first and the field lines that are close to the equator (field line represented by ‘L’ in Fig.1) experience the Alfvén wave perturbations later. Thus there is a phase lag of 2π/2 radians between the polar and equatorial solar activities. This reasoning that Alfvén wave perturbations reach first the poles and then the equator is consistent with the analysis of the sunspot butterfly diagrams [29], the observations of torsional oscillations on the surface [30, 31], the theoretical [4] and helioseismic inferences [32, 33] and, in the atmosphere [34].

Perturbations of the poloidal field structure in the convective envelope in turn perturb the embedded toroidal field structure and, the superposition of many such azimuthal perturbations attains a critical strength leading to the formation of the sunspots and due to buoyancy rise along the isorotational contours and reach the surface. For example, if one accepts the existence of such a steady part of the toroidal magnetic field structure with a strength \( B_r \), then the perturbations result in the creation of MHD waves whose amplitudes are \( \sim \delta B_r \). The superposition of many such MHD waves along the toroidal ring in turn leads to constructive interference and form the sunspots and, erupt towards the surface along the isorotational contours. As for the reversal of polarity, once sunspots are formed, they rise towards the surface in a particular latitude belt due to buoyancy and meridional flow transports remnant of the magnetic flux on the surface towards the poles and change the sign.

Due to the turbulence in the convective envelope, the amplitude of the Alfvén wave perturbations travel along the poloidal field (isorotational contours) will be considerably reduced near the surface. That means there is a need for constant forcing every 22 years near the center. Hence, it is not surprising that the resulting 11 year solar cycle and the activity phenomena on the surface can be considered as a forced and damped harmonic oscillator [35]. In this way, the third difficulty of the theory of MHD oscillations can be removed.

**Implications for the combined poloidal and toroidal field structure**

Some other consequences of having such a steady toroidal magnetic field structure in the convective envelope are: (a) perturbations to the thermal sound speed in the solar interior that contribute to the splitting of the even degree \( p \) modes [36, 37, 25]; (b) explanation for the recent discovery of ubiquitous horizontal magnetic field structure in the quiet-sun internetwork regions pervading every where in the photosphere as detected by Hinode satellite ([9] and references therein); (b) Alfvénic perturbations of the poloidal field structure ([4]; HG95) should yield the periods around 22 years and of the toroidal field structure should yield periods around 1-5 years.

**Physics of the 1-5 year quasi periodicities**

As for the steady toroidal field structure, the periods are computed from the relation \( T \sim L/V_A \), where \( T \) is the period of oscillations, \( L \) is the length scale of the field lines and \( V_A \) is the velocity of the Alfvén wave. In the case of the toroidal field structure, the length \( L \) is considered to be \( \sim 2\pi r \), where \( r \) is the radius of the ring.
along the azimuthal direction. For example, at 0.1 radius of the sun, the perturbation of the ring of toroidal field structure with intensity $10^5 G$ and density of 87 gm/cm$^3$ [38-39] yields a period of ~5 years. If we accept the model [3] of steady part of toroidal magnetic field structure (with a intensity ~ $10^4 G$ near base of the convection zone and ~ 1G near the surface) in the convective envelope and by taking the typical density values, the period of the oscillations vary from ~ 1.3 years near the base of the convective envelope to ~ of few months near the surface. These physical inferences imply that as the Alfven wave perturbations travel along different field lines (or along different isorotational contours) of the poloidal field structure and reach the surface from the pole to the equator, one would expect periodic phenomena at a particular latitude zone on the surface that is connected with periodic phenomena at a particular radius in the solar interior. To be precise, from the flux function $\phi(\vartheta) = 5.87 \sin^2 \vartheta - 1.59 \sin^4 \vartheta$

(from equation (19) of HG95; it is to be noted that $x=1$ is at the base of the convection zone and $x=1.43$ is observed surface; $\vartheta$ is co-latitude that increases from the pole to equator) on the surface, one can compute the intersection of different field lines on different latitudes. For example, for the flux values of 0.51 and 1.02 (B and C field lines of Fig 1 that deeply penetrate near the solar core around 0.1-0.2 solar radius) the latitudinal intersection is 73 and 65 deg heliographic latitudes, respectively and, for the field line 1 (with flux=3.7, in Fig.1) that penetrates close to base of the convection zone has a heliographic latitudinal intersection of 26 deg. Hence, from these inferences and with the poloidal field structure (between the field lines zone represented by A-C of Fig.1), one would expect near 5 year periodic phenomena, that originate in the beginning of the solar cycle and near 0.1 solar radius, should occur at the higher latitude zones on the surface. Similarly ~ 1.3 yr periodic phenomena that occur near the base of the convection zone (between the field lines represented by the symbol I-J of Fig.1), travel along field lines and reach the surface around the solar cycle maximum around 25 deg latitude (or co-latitude of 65 deg from the pole) zone on the surface. To conclude this subsection, in addition to 11 year periodicity (due to a weak poloidal field structure) in both the hemispheres, near 5-1.3 yr and ~ months periodicities should occur during certain phase of the solar cycle. From the observed analysis of different solar activity indices, let us examine in the following whether conclusion of this section is right or wrong.

Observations show that near 5 and 1.3 yr periodicities are indeed quasi-periodic and occur at different epochs (or at different latitude zones on the surface) of the solar cycle. For example ~ 5 yr quasi-periodicity is mainly detected in the high latitude zones ([40] and references there in). Although ~ 11 yr periodicity is dominant in the high latitude filaments [41], ~ 5 yr periodicity has a very low spectral power in their analysis.

As for the ~ 1.3 yr periodicity, it is detected in the sunspot data [42, 43], in the photospheric mean rotation [44], in the magnetic fields inferred from H-alpha filaments [45], in the large scale photospheric magnetic fields [46], in the green coronal emission [40], in the occurrence of coronal mass ejections [47], in the solar wind velocity, geomagnetic activity index Ap, and in the interplanetary magnetic field [43]. Spherical Harmonic Fourier (SHF) analysis [48, 49] of the magnetograms taken over 22 years shows the combined powers for the period of 22 yr (due to a weak poloidal field of ~ 1 G [50] and 1-5 years (due to a strong toroidal field of 104 – 105 G), respectively. From the helioseismic data, 1.3 yr periodicity is detected near the base of the convection zone ([51] and references there in). However, using the same helioseismic data, Antia and Basu [52] conclude that there is no 1.3 yr periodicity near the base of the convection zone. Further analysis by Basu and Antia [53] shows somewhat similar period as mentioned by Howe [51] but they did not consider it to be significant. Interestingly, as expected from this study, the post-2001 helioseismic data (see Fig.32 of Howe [49]) shows the disappearance of the 1.3 yr periodicity. More data analysis is required in order to confirm the physical inferences of this study.

**Alfven Wave Travel Times**

From the SHF analysis [48, 54, 55] of the Sun’s surface magnetic field, it is found that the axisymmetric terms of odd parity modes have nearly the same periodicity (~22 years). This indicates that the Alfven travel times may be approximately the same along different field lines of a steady magnetic field structure. In order to check the admissibility of such global oscillations, we have computed the Alfven wave travel times $T = \int ds / V_A$ along different field lines (that originate at the center and cut across the surface from pole to the equator), where $ds$ is a line element of the magnetic field structure and $V_A$ is the Alfven wave velocity. The Alfven wave travel times are computed in the following models by taking into account the real density variation in the sun: (i) the uniform field, (ii) the dipole field, (iii) a combination of the uniform and dipole fields, (iv) a combination of the dipole and hexapole fields embedded in a uniform field [56] and, (v) solution of a diffusion equation in an incompressible medium of constant diffusivity (HG95).

For the sake of comparison, all the models are assumed to have the same amount of magnetic flux with a nominal value of 1.5x10$^{22}$ Mx corresponding to a uniform field of ~1 G. It is found that the last two models yield the same period of 22 years. It is concluded that, owing to the regularity (without singularity) of the magnetic field structure at the center, the last model can be most likely the suitable geometrical magnetic field structure that sustains the ~22 years oscillations for all the field lines explaining the constancy of ~22 years of the observed SHF analysis of odd parity modes.

**Coupling of Long Period Poloidal and Toroidal MHD Oscillations and the Maunder Minimum type of activity**

Observational evidences show that the sun experienced the dearth of sunspot activity in the past evolutionary history. Yet there is no complete consensus among the solar community whether such grand minima are chaotic or regular. However, in the previous study [55], end of section 3), it was concluded that the
Maunder minimum type of activity is not chaotic and must be periodic with a period of ~ 100 years. Although most of the dynamo mechanisms ([9] and references therein) treat the long term variation of the solar cycle and activity phenomena as chaotic, based on previous studies ([57-58]; [35] and references therein), we consider the solar cycle and activity phenomena to be periodic.

\[ w_1 = w_0 \sqrt{1 - \left(\gamma / w_0\right)^2}, \]
\[ w_2 = w_0 \sqrt{1 + 2 \left(\gamma / w_0\right)^2 - \left(\gamma / w_0\right)^4}, \]
\[ w_T = 2\pi \left(\sqrt{V_{AP}^2 + V_{AT}^2} / \delta R\right) \]

is the coupling frequency due to poloidal and toroidal oscillations, \( \gamma \) is the dissipative factor, \( V_{AP} \) and \( V_{AT} \) are the Alfvén velocities due to poloidal and toroidal magnetic field structures and \( \delta R \) is the distance between the neighboring field lines. The first term in the RHS of equations (2) and (3) is the oscillation due to the poloidal magnetic field structure and the second term in the RHS of both the equations are the coupling of oscillations due to both poloidal and toroidal field structures with a coupling frequency \( w_T \).

In order to closely match with the 11 year solar cycle and the long term variation of the sunspot activity, the fundamental period due to the poloidal oscillations must be 22 years (or frequency \( \omega_0 \) is \( \sim 0.286 \) rad/yr), the dissipation factor \( \gamma \) must be 0.185 and the coupling frequency \( \omega_c \) should be 0.11 rad/yr. It is interesting to note that the theoretical dissipation factor \( \gamma \) of 0.185 is almost the same as the dissipation factor of 0.186 obtained from the observed solar cycles [35]. The simulation of magnetic energy (square of amplitude of either poloidal or toroidal oscillations with arbitrary and equal amplitudes of \( a_0 \) and \( a_1 \) of such coupled oscillations with respect to the time span of 500 years (Fig.2) shows that oscillations of the poloidal field with a fundamental period of 22 yrs excite the toroidal field oscillations such that the toroidal field structure oscillates in consonance with the poloidal field oscillations resulting in the coupling of poloidal and toroidal oscillations that reproduce the observed cyclic periodicities of 11 and 100 yrs with a very deep minimum around 350 years when both the strengths of poloidal and toroidal oscillations have very low amplitudes. The paleoclimatic records show that during the Maunder minimum period although the sunspot activity was practically absent, the 11 year activity due to geomagnetic indices [60] and solar proxy records [61-63] was present. As the activity of the geomagnetic indices [64-66] and the solar proxy records are considered to be mainly due to the solar polar magnetic activity, the simulation of long term solar activity due to the poloidal oscillations in Fig.2 shows also normal activity during the Maunder type deep minimum activity confirming the observations.

Conclusions

Two theoretical models on the genesis of solar cycle and activity phenomena, viz., turbulent dynamo and MHD oscillations mechanisms are critically examined. Several difficulties and limitations in their application to the solar cycle and activity phenomena are presented. The seminal work of Alfven on the theory of solar cycle is revisited and its limitations are presented. New ideas on the genesis of the solar cycle and activity phenomena and their long-term variations are presented. Overall conclusions of this study are: (i) the most likely existence
of a combined poloidal and toroidal magnetic field structure in the solar interior is proposed. (ii) In the frame work of MHD oscillations mechanism, the genesis of solar cycle and activity phenomena is discussed, (iii) implications of having such a combined poloidal and toroidal field structure in the solar interior are discussed and the physics of the 1-5 yrs solar quasi periodicities is explained. (iv) for different models of the poloidal magnetic field structure in the solar interior, Alfvén wave travel times are computed and it is found that among all these models, HG95 model yields almost similar periods of 22 yrs for all the field lines explaining the constancy of ~ 22 yrs period in the SHF analysis of odd parity modes and, (v) Maunder minimum type activity is explained to be due to coupled long period poloidal and toroidal MHD oscillations.

Acknowledgements

Author is thankful to unknown referee for the useful comments, to Prof. Stenflo and Dr. Gopalswamy for the useful discussions. Author is also thankful to Dr. Katya Georgieva and NASA for providing full financial support for attending the conference.

References