An Empirical Model for Determination of the Cosmic Ray Spectra

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The high-energy charged particles entering the Earth's environment from interplanetary space have the significant effect on the space weather. We propose an improved model, which generalizes the differential D(E) and integral D(>E) spectra of galactic (GCR) and anomalous cosmic ray (ACR) protons and heavier elements during the 11-year solar cycle. The model takes into account the cosmic ray (CR) modulation by the solar wind and numerous solar and heliospheric events during different levels of solar activity. Modulated energy spectrum of galactic cosmic rays is compared with force field approximation. The difference is in the order of 1.5 %. The model solutions are compared with IMAX-92, CAPRICE-94 and AMS-98 measurements. This computed analytical model gives practical possibility for investigation of experimental data from measurements of galactic cosmic rays and their anomalous component. The obtained parameters are used for determination the profiles of the ionization in the ionosphere and middle atmosphere.

Introduction

The history of the cosmic ray (CR) modulation studies is as long as the history of regular CR observations. Initially, these studies were strictly tied to the Earth. Even now, in the epoch of Voyager and Ulysses when successful cosmic ray experiments have been flown into the distant heliosphere, the Earth is still the best place to investigate CR modulation [1].

The solar wind speed should be of major importance for CR modulation since it determines two components of the modulation mechanism: convection and adiabatic energy changes. But the cosmic ray modulation is a result of numerous solar and heliospheric events occurring both simultaneously and consecutively, and affecting different energies [1]. It is best to consider modulation of anomalous cosmic rays (ACRs) and galactic cosmic ray (GCRs) together in a single model. We propose an improved semi – empirical model, which generalizes the differential *D(E)* and integral *D(>E)* spectra of galactic and anomalous cosmic ray protons and heavier elements during the 11-year solar cycle using a simple law.

Modeling cosmic ray differential spectra

The observed CR spectrum can be divided into the following five intervals: I (E = $3{\cdot}10^6$ – 10^{11}GeV/n), II (E = $3{\cdot}10^2$ – $3{\cdot}10^6$ GeV/n), III (E = 30 MeV/n – $3{\cdot}10^2$ GeV/n), IV (E = 1 – 30 MeV/n), V(E = 10 KeV/n – 1 MeV/n), where E is the kinetic energy of the particles [2, 3].

Some methods exist for calculating ionization by relativistic particles in CR intervals I, II and III.

The energy interval 1 MeV - 100 GeV includes both anomalous/solar and galactic cosmic rays. Up to 7-10 MeV at solar minimum and up to 20-30 MeV at solar maximum cosmic rays have been argued to be predominantly of anomalous and solar component, respectively [4].

Proposed computed analytical model gives a practical possibility for computation of the ionization profiles for different latitudes and different levels of solar activity and investigation of experimental data from measurements of galactic cosmic rays and their anomalous component.

Primary differential energy spectrum of protons and other groups of cosmic ray nuclei in energy interval 1 MeV-100 GeV is presented with a single law [3]:

$$D(E)=\left(\frac{1.9\times10^4 P(E)^{-2.78}}{1+0.4866P(E)^{-2.51}}\right)\left(1+\frac{\alpha}{E}\right)^{-\beta}\times$$
$$\times\left\{\frac{1}{2}+\frac{\tanh[\lambda(E-\mu)]}{2}\right\}+$$
$$+\frac{x}{P(E)^{y}}\left\{\frac{1}{2}-\frac{\tanh[\lambda(E-\mu)]}{2}\right\}, \qquad (1)$$

where *P(E)* is the rigidity *(GV)* taken as a function of energy *E* (GeV). The quantities α (GeV) and β are modulation coefficients, which are related to the solar activity levels. The anomalous cosmic ray spectrum is determined by coefficients x (GeV^y/m^{2}sec·ster·GeV) and y at different heliosphere conditions. The spectral indexes β and y are dimensionless quantities. The members with *tanh* are smoothing functions between galactic and anomalous cosmic rays. The dimensionless parameter λ=140. The physical meaning of μ (GeV) is the crossing energy at which the differential spectra of GCR and ACR contribute to the half of their values. The unit of differential intensity *D(E)* is [part/m^2·sec·ster·GeV].

In equation (1) the first addend gives the main contribution between the energies from several MeV to 100 GeV. This term describe the galactic cosmic rays. The differential spectrum *D(E)* in the energy range 1.8 MeV ÷ 30 MeV is determined by the second addend and it is related with the behaviour of anomalous/solar component.

In (1) we use the local interstellar spectrum according to Burger et al. [8]:

$$D_0(E)=\frac{1.9\times10^4\,P(E)^{-2.78}}{1+0.4866P(E)^{-2.51}}$$

The effect of the solar cycle on ACR observations at 1 AU is extreme. There is an increasing presence of very large solar energetic particle (SEP) events in which the intensities can rise above the ACR intensities by many orders of magnitude. At solar maximum, quiet periods are rare and limited in duration. For this reason, nearly all ACR measurements have been made near solar minimum, and the behaviour of the ACRs at solar maximum has not been known [5, 6].

Correctly accounting for solar modulation of the ACRs as they are transported inward from the termination shock and distinguishing the effects of acceleration and modulation is a difficult task [7]. In our model we take into account simultaneously the effects of acceleration and modulation using a power law in rigidity.

Thus modulated CR spectrum can by used for computation of the ionization profiles for different levels of solar activity. The electron production rate q ($cm^{-3}\ s^{-1}$) as a function of height h *(km)* for particles of type i from the cosmic ray composition is given by the following expression [9, 10, 11]:

$$q_i(h) = \frac{2\pi}{Q} \int_{E_i}^{\infty} \int_{\theta=0}^{\pi/2+\Delta\theta} D_i(E,\ h,\ \theta) \left(\frac{dE}{dh}\right)_i \sin\theta\, d\theta\, dE \qquad (2)$$

where Q (eV) is the energy required for the formation of one electron-ion pair and depends on the atmospheric composition; $D_i(E)$ is the differential spectrum of the particles; E is their kinetic energy; E_i is the energy (GeV/nucl), which corresponds to the geomagnetic cut-off rigidity R_C(GV); dE/dh represents the ionization losses of the penetrating CR particles, expressed by Bohr-Bethe-Bloch formula; θ is the angle towards the vertical; $\Delta\theta$ takes into account that at a given height the particles can penetrate from the space angle (0º, θ_{max}= 90º + $\Delta\theta$), which is greater than the upper hemisphere angle (0º,90º) for flat model. Velinov [10] and Velinov et al. [11] give the law of transformation of the spectrum $D(E)$ at different altitudes by penetration of CR particles in the atmosphere. Under 20-25 km in this model must be included the nuclear interactions. But for the ionosphere investigations this is not necessary. Equation (2) gives the permanent CR ionization in the Earth' atmosphere. The other particles (magnetospheric or trapped) have contribution only at high latitudes.

The average energy loss of particles (mass A, charge Z) from interstellar space to 1 AU is given by the potential energy $\Phi = \varphi Z/A$, where φ is the solar modulation parameter in MV. The differential intensity $D(E,\Phi)$ (part/m^2·sec·ster·MeV) in terms of the solar modulation parameter is [12]:

$$D(E,\Phi) = D_0(E+\Phi)\frac{E(E+2E_0)}{(E+\Phi)(E+\Phi+2E_0)} \qquad (3)$$

where E is the kinetic energy in MeV per nucleon, $D_0(E+\Phi)$ is the interstellar flux.

In this work the modulated differential energy spectrum of galactic protons and helium nuclei is compared with force field approximation. The coefficients α and β are determined at different values of solar modulation parameter φ.

The measurements with the IMAX-92 [13], CAPRICE-94 [14] and AMS-98 [15, 16] experiments are used at numerical solutions of the model equation (1) for galactic cosmic ray component:

$$D(E) = \frac{1.9\times10^4 P(E)^{-2.78}}{1+0.4866 P(E)^{-2.51}}\left(1+\frac{\alpha}{E}\right)^{-\beta} \qquad (4)$$

The coefficients α and β are determined by Levenberg-Marquardt algorithm [17], applied to the special case of a least squares. The described program is realized in algorithmic language C++.

Results

The difference between force field approximation and our model is about 1.5 %.

In Fig.1 are shown the results from the differential energy spectrum $D(E,\Phi)$ (equation 3) of galactic protons for four values of modulation parameter: φ = 400, 550, 700 and 1200 MV. In Fig. 2 these results are given for galactic helium nuclei.

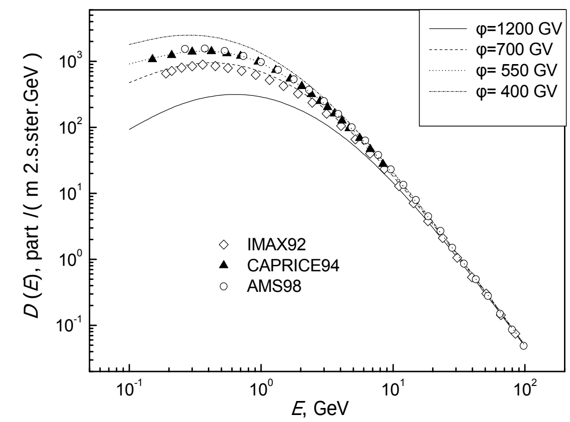

Fig. 1. The differential spectra D(E) of GCR protons for modulation levels φ = 400, 550, 700 and 1200 MV. Measurements with IMAX-92 [13] are in the vicinity of $\varphi \approx$ 700 MV, CAPRICE-94 [14] and AMS-98 [15] of $\varphi \approx$ 550 MV.

These spectra are compared with the measurements for the periods of solar maximum ■ – IMAX-92 [13], and near to solar minimum (two years before the solar minimum 1996) ▲– CAPRICE-94 [14] and • – AMS-98 [15, 16]. Measurements with IMAX-92 are near to $\varphi \approx$ 700 MV, while CAPRICE-94 and AMS-98 to $\varphi \approx$ 550 MV. By all model calculations galactic part of model equation (1) is used, i.e. equation (4), because these experiments are related only to galactic cosmic ray component.

In Tables 1 and 2 the values of coefficients α and β and the corresponding values of χ^2_n for experiments IMAX-92, CAPRICE-94 and AMS-98 are given for galactic protons and helium nuclei, respectively. It is seen from Tables that α value increase from solar minimum to solar maximum, while β decrease. In this case we obtain the right shift of the spectral maximum as well as decreasing of it's amplitude with increased solar activity.

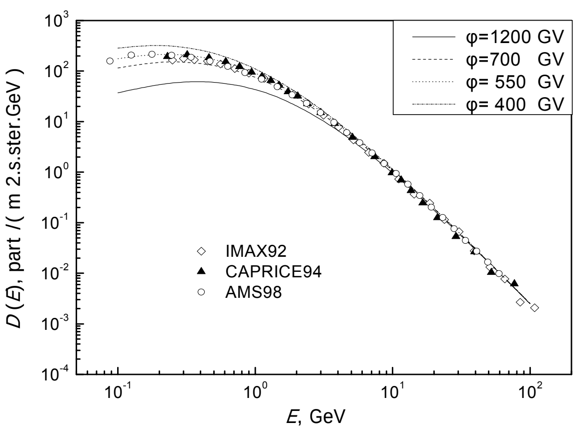

Fig. 2. The differential spectra D(E) of GCR helium nuclei for modulation levels φ = 400, 550, 700 and 1200 MV. Measurements with IMAX-92 [13] are in the vicinity of $\varphi \approx$ 700 MV and CAPRICE-94 [14] and AMS-98 [16] of $\varphi \approx$ 550 MV

TABLE 1

Coefficients α, β and χ^2_n of protons for the experiments IMAX-92 [13], CAPRICE-94 [14], AMS-98 [15]

Experiments	IMAX-92	CAPRICE-94	AMS-98
α	3.872095	2.277076	1.947788
β	1.179442	1.206174	1.239947
χ^2_n	0.278058	2.242491	0.240423

TABLE 2

Coefficients α, β and χ^2_n of helium nuclei for the experiments IMAX-92 [13], CAPRICE-94 [14], AMS-98 [16]

Experiments	IMAX-92	CAPRICE-94	AMS-98
α	1.927907	1.938468	2.464154
β	0.834376	0.726815	0.695075
χ^2_n	0.603408	4.151563	0.300887

Conclusion

This computed model gives a practical possibility for presentation of experimental data from CR measurements by means of analytical expression. The expression (1) is basic for determination of ionization profiles (2) in the planetary ionospheres. In such a way our formula (2) modeled the CR differential spectra *D(E)* in whole energy interval 0.001–100 GeV. The other models are numerical or represent data bases. They relate only to some energy intervals.

From electron production rates we can determine the electron density and electrical conductivities in the planetary ionospheres. This is very important for the physics of the ionospheres and solar-planetary relationships. GCR influence significantly, through atmospheric conductivity, the electric fields and currents in the Earth environment, generated by thunderclouds [18, 19] and by thunderstorm activity [20]. GCR cause an exponential increase of the conductivity with altitude, and thus orientation of the conductivity currents generated by a thunderstorm within a relatively narrow (<100 km) vertical tube into the magneto/ionosphere [18, 19]. Since the conductivity in the troposphere and the lower stratosphere is determinative for the electric resistance between the ionosphere and the ground, GCR can play a role of an important factor, which controls the parameters of the global atmospheric electric circuit and of the ionospheric potential variations.

However, the expression (2) takes into account only the electromagnetic interactions of CR with the substance of the atmosphere. That is why (2) is valid only in the planetary ionospheres and not in the whole atmospheres. The mean path of the nuclear interactions for the protons is about 70 g·cm^2. That means the ionization model (2) works well above 12 - 15 km for terrestrial atmosphere [11]. Under that altitude the nuclear interactions enforce and the secondary particles must be taken into account (see [21]). But the test of the model gives real values of the ionization rates, even at 25 and 20 km. That means the electron production rate q does not ''feel'' if the ionization factor is primary or secondary cosmic rays. Below 12 km the effect of Pfotzer maximum [22] begins and the model (2) is not valid.

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