Main Features of Quasi-Electrostatic Fields in Atmospheric Regions due to Lightning Discharge

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The most important features of the strong quasi-electrostatic fields, generated in the region ionosphereground after a single lightning discharge, due to succeeding redistribution of capacitive spatial charges, are studied in this work. The investigation of these fields is of great importance, particularly since they are considered to be responsible for generation of red sprites in the mesosphere and lower ionosphere over thunderstorms. As experimental measurements show, these quasi-electrostatic fields can cause, more often than sprites, electron heating and conductivity modifications in the mesosphere and ionosphere as well. The temporal behavior and relaxation time of the quasi-electrostatic fields, as well as their spatial extent and orientation are studied. For this purpose an analytical model based on the Maxwell equations under conditions of curl-free electric field is proposed. Such conditions are fulfilled short (less than a millisecond) after beginning of a lightning discharge and last until another discharge. Isotropic conductivity in the region of interest is assumed. Computations are made for the time-course of the quasi-electrostatic fields up to the lower ionosphere. The dynamics of spatial charges in this region, responsible for the quasi-electrostatic fields, is analyzed also. The quasi-electrostatic fields at an altitude observed reach their maximum at a time close to the local relaxation time. Up to the ionosphere this maximum is of an order of V/m per 1 Coulomb of a charge transported by lightning. The field decreases immediately after reaching its maximum in the mesosphere; however, at stratospheric and tropospheric altitudes its maximum forms a plateau, which becomes wider at lower heights. The relaxation of the quasi-electrostatic field at altitudes above 40 km is essentially slower than the local relaxation time of an electrical charge.

Introduction

It is well known that strong quasi-electrostatic fields (QESF) are generated in the middle atmosphere after a tropospheric lightning discharge (for example [1, 2]). Their source is the so-called 'Wilson monopole', formed by charges, which are generated by a thundercloud charge Q_0 and have a screening effect for the electric field of Q_0 . These charges are located mainly close around the thundercloud charge. When this thundercloud charge is removed by lightning, the electric fields due to the "Wilson monopole" remain relatively unscreened for a time period comparable to the relaxation time in the region close above it. During this time period these electric fields penetrate upward in the atmosphere without strong attenuation, typical under DC conditions. These strong fields are an important source of electron heating in the mesosphere and the ionosphere. A series of lightning discharges during a long-lasting thunderstorm can lead to considerable modifications of the conductivity in these regions, and even can provoke electron density variations in the ionosphere, experimentally established by some authors [3]. Also, QESF above large thunderstorms are considered as an initiator of red sprites [4] because of their possibility to cause conventional breakdowns. According to another possible mechanism of sprites, these are caused by a flux of run-away electrons with relativistic energies, accelerated by QESF [5]. Regardless of the actual mechanism, a comprehensive and detailed analysis of temporal and spatial behavior of QESF and their features is necessary, in order to answer a series of questions related to physics of red sprites and other effects in the mesosphere and ionosphere. All this shows that the investigation of quasielectrostatic fields due to lightning is of great importance. A series of studies [6, 7] indicate a complex temporal and spatial behavior of QESF. This behavior is significantly influenced by many factors. Such are the thunderstorm and lightning discharge parameters: the discharge type, its time course and characteristic time constant, the height, magnitude, spatial distribution and dimension of the removed thundercloud charge, the parameters of long-lasting currents, etc. Other factors relate to the atmospheric conductivity. Having in mind the great variability of most of these factors, as well as their difficult experimental observation (separately or simultaneously), one reveals the complexity of the problem of QESF investigation. The theoretical results recently obtained have to be developed further, in order to comprise better this complexity, and new studies are needed to enrich knowledge of QESF features.

The goal of this work is to demonstrate some important features of spatial and temporal behavior of the quasielectrostatic fields and how they are related to both the discharge time and the local relaxation time and to the features of the atmospheric conductivity profile. An analytical model is developed for the goal of this study, which is based on the system of Maxwell equations for quasi-static conditions. The quasi-electrostatic fields are compared with the steady-state fields, which take place in the pre-discharge period. They differ by a factor, which sharply increases with altitude and exceeds three orders at altitudes close to 80-85 km. This shows that the peak QESF intensities decrease with the altitude much slower than the breakdown electric field. This is a key feature for the possibility of QESF to initiate red sprites by a conventional breakdown. The time of QESF relaxation is evaluated as dependent on altitude: it is shown that it is much larger than the local charge relaxation time. In order to explain these features better, the behavior of QESF and of the spatial charges created due to the capacitive atmospheric properties, are studied together.

Analytical Modeling

Presentation of a thundercloud charge removal by lightning

The decrease of a thundercloud charge of initial amount Q_0 due to a lightning discharge which begins at time t=0 is exponential according to the following formula, used also by [5]:

$$Q(t) = Q_0 \exp(-t/\tau_L), \qquad (1a)$$

where $\tau_{\rm L}$ is the characteristic discharge time. Other time courses of a discharge are discussed for example by *Pasko et al.*[2], *Cho and Rycroft* [7], etc. The remaining charge Q(t) at time *t* is considered as distributed at a horizontal plane with surface density:

$$\rho_Q(r,t) = Q(t)/R_Q \exp(-r/R_Q), \qquad (1b)$$

where R_Q is the effective horizontal radius of the charge. Eq.(1b) is written in cylindrical coordinates (r, φ, z) such that z is an upward oriented axis through the charge with z=0 at the ground surface.

Equation and initial and boundary conditions

We develop a model under assumption for a curl-free electric field. The validity of this assumption is discussed in some works [8, 9], where it is shown that it is fulfilled about 1 ms or little less time after the beginning of a lightning discharge – at earlier times the influence of the magnetic component of the electromagnetic field can not be neglected. Under such assumption the quasi-electrostatic field **E** is characterized by an electric potential U.

In order to obtain **E**, the well-known continuity equation for the density of the Maxwell current $\mathbf{j} = \mathbf{j}_{C} + \mathbf{j}_{D}$ is solved, similarly to [5]:

$$\nabla \cdot \mathbf{j} = 0 \,. \tag{2}$$

Here $\mathbf{j}_C = \sigma \mathbf{E}$ and $\mathbf{j}_D = \varepsilon_0 \partial \mathbf{E}/\partial t$ are densities of the conductivity and displacement currents, respectively; $\sigma = \sigma(z)$ is the atmospheric conductivity at altitude z, and ε_0 is the dielectric constant of the air. The region of Eq.(1) is between the ground and an upper boundary Z_B located in the ionosphere. In this work $Z_B=100$ km, however results up to only z=90 km are considered in order to avoid the impact of the boundary. Isotropic conductivity (characterized by a scalar) is assumed in the region of interest. Although this is not true in general above 70 km, such simplification does not influence significantly the vertical component of the electric field $E_z = -\partial U/\partial z$, but only the radial one, $E_r = -\partial U/\partial r$. Since the interest here is to the maximal QESF reached at r=0, where $E_r=0$, the component E_r is not studied in this work.

An initial condition at t=0 (the lightning discharge beginning) is used. It is given by the distribution of the total system of charges (thundercloud charges, as well as spatial charges of capacitive nature). Under an assumption for DC conditions before the discharge, the spatial charge density ρ is the following:

$$\rho(t=0) = -\varepsilon_0 E_z^0 / h \,. \tag{3a}$$

Here E_z^0 is the electric field, which is established under steady-state conditions by the thundercloud charge Q_0 before the discharge. The initial condition total charge density is written as:

$$\rho_{\text{tot}}(t=0) = \rho_0(t=0) + \rho(t=0).$$
 (3b)

The following boundary conditions are defined:

$$U=0$$
 at the ground ($z=0$) (4a)

U=0 at the upper boundary, $z=Z_{\rm B}$ (4b)

$$E_{z}(r, Z_{Q}+0, t) - E_{z}(r, Z_{Q}-0, t) = \rho_{Q}(r, t)/\varepsilon_{0}.$$
 (4c)

The first condition (4a) means that the surface can be considered as a perfect conductor related to the adjacent air. The second boundary condition (4b) is more disputable. Following to [10], the electric fields in the ionosphere can be very sensitive to the boundary condition type at the upper boundary of the modeled region. Condition (4b) claims that at times of interest, i.e. at $t \ge 10^{-4}$ s, the variations of the potential at altitudes with relaxation time $\tau_{\rm R} \ll 10^{-4}$ s can be neglected. This condition is valid when the ionospheric potential is not taken into account (the fair-weather electric fields can be added independently due to the model linearity). On the other hand, a boundary condition at $z=Z_B$ of types $E_z=0$ or $E_z \rightarrow 0$ by $z \rightarrow \infty$ can not correctly take into account the large and fast variations of the electric fields in the region around the boundary. At last, condition (4c) reflects the impact of the thundercloud electric charge to the electric field Q(t).

Conductivity profile

In order to obtain an analytical solutions to Eq.(2), a special approximation of the actual conductivity profile by a stepwise function by the altitude z is used. This approximation is of the following form:

$$\sigma(z) = \sigma_i = \text{const} \quad \text{when} \quad z_{i-1} < z \le z_i \tag{5}$$

Here $[z_{i-1}, z_i]$, i=1...,m define *m* succeeding horizontal layers, which cover the model region. The approximated conductivity is a constant within each, so that $\sigma_i/\sigma_{i-1} \leq C_{\sigma}$. The factor $C_{\sigma}>1$ has to be small enough for better accuracy of the conductivity approximation. At layer boundaries where conductivity is discontinuous, continuity of the vertical component j_z of the Maxwell current density is required.

A nighttime atmospheric conductivity profile at middle latitudes by quiet conditions and moderate solar activity, obtained in [10] as combination of many experimental measurements and some theoretical results, is used in this work. This profile between 0 and 100 km is shown in Fig.1 by crosses, after a slight modification. It is approximated by a step-wise function defined by (5) by $C_{\sigma}=1.35$, shown in Fig.1 by a solid line. The relative deviation of the approximated from the original curve does not exceed 1.162. Such high accuracy of the conductivity profile approximation is needed since the quasi-electrostatic fields at high altitudes are highly sensitive to even slight conductivity profile variations at some lower altitude z. As shown further, this sensitivity is realized as a modification of QESF at time t equal to the relaxation time $\tau_{\rm R}$ at the altitude of conductivity modification. Similar effect is seen in [6], however it concerns the total Maxwell current to the ionosphere.

Cases of moderate or weak lightning discharges are considered in this work, whose quasi-electrostatic fields do not cause significant modification to the conductivity profile.

Solutions

Analytical solutions are obtained for the vertical (E_z) and radial (E_r) components of QESF in the form:

$$E_{z} = \int_{0}^{\infty} J_{0}(rk) \partial F_{i}(k, z, t) / \partial z \, dk ; \qquad (6a)$$

$$E_r = \int_0^\infty k J_1(rk) F_i(k, z, t) dk$$
(6b)

where J_0 and J_1 are the Bessel functions of first kind and of orders 0 and 1. Functions F_i are different for each conductivity layer $\underline{i} = 1, ...m$ and are determined as a solution of a system of ordinary differential equations for $\partial F_i / \partial t$ defined by the initial (3) and boundary (4) conditions and the requirement for continuity of the vertical component j_z of the Maxwell current density at the layer boundaries $z=z_i$. The analytical solution is given in more details in [5].

Behavior of Quasi-Electrostatic Fields and Spatial Charge Density

The model is used to study the temporal and spatial distribution of QESF due to a single cloud-to-ground lightning discharge in the height region 40-90 km and within a time period of 1 s after the discharge onset at t=0. The thundercloud charge, which is destroyed by lightning, is initially distributed at a horizontal plate at altitude $z_0 = 10$ km according to (1), with an effective radius $R_Q = 300$ m and by initial magnitude $Q(t=0) = Q_0$. The removal of this charge is according to Eqs.(1) by characteristic time $\tau_{\rm L} = 1$ ms. It is assumed that steady-state conditions take place before the discharge, i.e. the thundercloud and spatial charges variations are slow enough so that the displacement currents can be neglected. The conductivity profile and its approximation by (5) (Fig.1) are used. The demonstrated results are for the positive initial charge $Q_0 > 0$. In the opposite case the studied characteristics will change their signs to the opposite ones.

The time course of the vertical component $|E_z(t)|/Q_0$, normalized with respect to the initial charge Q_0 , is shown in Fig.2. Computations are made for different altitudes z=40, 50, 60, 70, 80, 85 μ 90 km, by r=0, where $|E_z|$ has a maximum by fixed altitude and time, and $E_r=0$. The steady-state electrical field above the thunderstorm at t=-0 has a positive vertical component $E_z^0>0$. During the discharge this component changes its sign to a negative at time $t_{Ez=0}(z)$, which depends on the altitude z. This change of the sign is due to the impact of the negative 'Wilson monopole', which remains unbalanced with the removal of the base thundercloud charge Q_0 .

In order to explain better the results in Fig.2, the behavior of the spatial charges density is studied in parallel, since QESF are controlled by their dynamics. Their behavior is shown in Fig.3, where each curve is for the normalized spatial charge density $\rho(r, z, t)/Q_0$ at fixed altitude z and r=0, where ρ reaches its maximum at z. At t=0 the spatial charges above the thundercloud charge Q_0 are negative ($\rho^0 < 0$), according to Eq.(3a). After the lightning discharge beginning, a region with positive charges is formed above the region of negative ones. It spreads downward with time. The curves in Fig.3 relate to a time period, when $\rho > 0$ at the proper altitude. These newly formed spatial charges are screening charges for



Fig. 1. Conductivity profile in the atmospheric region 0 – 100 km at middle latitudes at night according to Hale (1984) (noted by crosses) and the approximation by a step-wise function applied (solid line).



Fig. 2. Time-course of the normalized quasi-electrostatic field $|E_z(t)/Q_0|$ at altitudes z=40, 50, 60, 80, 85, and 90 km by r=0 due to a removal of a charge Q_0 at altitude $Z_Q=10$ km, by $R_Q=0.3$ km, and by discharge time $\tau_L=1$ ms. Conductivity profile in Fig.1 is used.

the electric field generated by the negative "Wilson monopole".

Fig.2 shows that at each altitude z when $|E_z|$ first increases, and reaches its peak value $|E_{zMAX}|$ at time $t=t_{MAX}(z)$, which depends on the altitude z (here and further index 'MAX' is for a maximum by time when z and r are fixed). After that, when $t>t_{MAX}(z)$, the vertical component $|E_z|$ diminishes asymptotically by time. At higher altitudes $z |E_{zMAX}(z)|$ is smaller and is reached sooner after the lightning discharge beginning, $(t_{MAX} \text{ is smaller})$. On the other hand, it is much bigger than the DC field $E_z^0(z)$ at the same altitude before the discharge. The reason in the case of higher altitude z is that no essential charge is formed yet at $t=t_{MAX}$ between z and z_Q , where the 'Wilson monopole' is located (seen in Fig.3), and thus screening effect is weak. On the other side, the increase of t_{MAX} at lower heights is caused by the fact that the decrease of $|E_z(z)|$ by $t > t_{MAX}$ is due to the screening charges, which are formed first at higher altitudes (Fig.3). After $|E_{zMAX}(z)|$ is reached, a peak of the spatial charges $\rho(z)$ at the same height is reached as well. This peak is at $t_{\rho MAX} \approx \tau_R(z)$, while $|E_z(z)|$ is already decreasing because of the positive screening charges already created below z. The QESF feature just discussed means that a slow decrease of $|E_{zMAX}(z)|$ with altitude is typical in the region up to $z_{LR}=82$ km, where the relaxation time $\tau_R(z_{LR})=\varepsilon_0/\sigma(z_{LR})$ is smaller than the lightning discharge time τ_L for the used conductivity profile. Above this region the decrease of $|E_{zMAX}|$ with altitude is much faster, and is similar to the respective decrease of the DC electric field. As a result, the maximum intensity of quasi-electrostatic fields at z>82 km is bigger than the intensity of the DC field before lightning by a factor of 3 orders and more (Table 1). In the stratosphere this factor is tens of times. The obtained results show that a typical lightning discharge with a charge moment change of hundreds of *Coulomb.km* QESF reach hundreds of V/m in the stratosphere, tens of V/m in the mesosphere, and several V/m at z=85 km.

TABLE 1. Main Characteristics of QESF Behavior at Different Altitudes by r=0

Altitude z, km	40	50	60	70	80	85	90
$ E_{z \text{ MAX}}/Q_0 $, V/m/C	7.44	3.69	2.44	1.77	1.02	0.128	1.2×10^{-3}
$ E_{z \text{ MAX}} / E_z^0$	16.8	40.7	126	187	277	1285	1490
$t_{MAX}(z)$, s	0.032	7.7×10^{-3}	3.5×10^{-3}	3.3×10^{-3}	2.5×10^{-3}	3.3×10^{-4}	2.5×10^{-5}
$t_{1e}(z)$, s	5.8×10 ⁻⁴	5.2×10 ⁻⁴	4.2×10^{-4}	4.1×10^{-4}	3.5×10 ⁻⁴	4.8×10^{-5}	-
$t_{2e}(z)$, s	0.54	0.14	3.5×10^{-2}	2.9×10^{-2}	1.5×10^{-2}	1.7×10^{-3}	1.2×10^{-3}
$ au_{\mathrm{R}}(z)$, s	0.2	7×10^{-2}	1.8×10^{-2}	1.2×10^{-2}	6.0×10 ⁻³	1.4×10^{-4}	1.1×10 ⁻⁶
t_{QESF} , s	2.1	2.2	2.4	2.7	3.2	0.25	1.2×10^{-2}

This is a key QESF feature for their possibility to initiate a conventional breakdown (by avalanches of electrons with thermal energies) in the mesosphere and lower ionosphere. In order to illustrate the possibility of a breakdown, in Fig.4 the height dependence of $|E_{zMAX}(z)|$ by $Q_0=100$ Coulomb (solid curve) is compared with the profile of the breakdown electric field E_K (dashed curve), which gives the threshold electric field needed to be applied in order to cause a conventional breakdown (Cho). E_K is proportional to the density of the neutral atmosphere, which is derived here from MSIS-90 for



Fig.3. Time-course of the normalized spatial charge density $|\rho/Q_0|$ at altitudes z=40, 50, 60, 80, and 85 km by r=0 and the same conditions as in Fig.2.

the summer and the same conditions as for conductivity profile used. It is seen from Fig.4, that up to $z=82 \text{ km } |E_{zMAX}|$ decreases with altitude slower than E_K so that by large enough initial charge Q_0 one can reach, with the height increase, to a region, where $|E_{zMAX}|$ becomes bigger than E_K and a breakdown is possible. Although this comparison is rather rough, since strong QESF cause first modifications (a decrease) of conductivity [2], and self-consistent analysis is necessary in such case for better accuracy, it is rather indicative, having in mind, that QESF will be even bigger due to diminished conductivity.

At low altitudes the time course $|E_z(z, t)|$ forms a 'plateau' around its maximum $|E_{zMAX}(t)|$, which becomes wider at lower heights (Fig.2). The explanation is that the relatively slow dissipation of the negative 'Wilson monopole' below the observed altitude z is compensated, for some time period, by an increase of positive spatial charges in a region above z. This period begins with the displacement of the thundercloud charge Q_0 by $t \sim \tau_L$ and finishes when $\rho(z)$ reaches its maximum by $t \sim \tau_{R}(z)$. The "plateau" is wider at lower z, since there $\tau_L \ll \tau_R$. It is missing at altitudes where $\tau_L \gtrsim \tau_R$. In order to do a comparative analysis of temporal characteristics of QESF, we define for boundaries of the 'plateau' as times t_{1e} and t_{2e} ($t_{1e} \le t_{MAX} \le t_{2e}$) from the condition that $t_{1e} \le t \le t_{2e}$ if $|E_z(t)| > 1/e |E_{z \text{ MAX}}|$. These are given in Table for the altitudes observed, as compared to $\tau_{\rm R}$. In Fig.5 time constants $t_{\rm MAX}$ (solid curve), and t_{1e} , t_{2e} (tiny dashed curves) are presented as functions of height z together with the relaxation time $\tau_{\rm R}$ (irregularly dashed curve with circle marks) and the characteristic discharge time $\tau_{\rm L}$ (irregularly dashed line without marks). We see that the height variations of t_{MAX} and t_{2e} are similar in the region with relaxation time bigger than



Fig.4. Profiles of the peak QESF (solid curve) generated under the same conditions, as in Fig.2, for $Q_0=100$ Coulomb, and the breakdown electric field E_k (dashed curve)



Fig. 5. Profiles of temporal characteristics t_{MAX} , τ_{1e} , t_{2e} , τ_R , compared with the lightning discharge time τ_L , under the same conditions, as in Fig.2.

the lightning discharge time, i.e. up to z_{LR} =82 km, and the relation $\tau_L < t_{MAX} < \tau_R$ is valid. Above z_{LR} , where the relaxation time becomes smaller than the lightning discharge time, the inverse relation is valid: $\tau_L > t_{MAX} > \tau_R$.

An important QESF feature, seen by Fig.1 and Table 1, is that at altitudes of interests QESF exists considerably longer than the relaxation time of an electric charge. In Table 1 t_{QESF} is the time period, when $|E_z|$ is bigger than the initial DC field E_z^0 . It is seen that in the ionospheric heights t_{QESF} is bigger than τ_R by several orders.

Conclusions

The following main conclusions are derived from results obtained.

1. The behavior of the quasi-electrostatic fields is determined, in general, by the following two factors:

(*a*) With time increase, when $t \le \tau_L$, the balance of the total system of electric charges in the atmosphere is stronger disturbed, and the domination of electric fields generated by charges which form the 'Wilson monopole' above ones due to the basic thundercloud charge, becomes stronger, so that it leads to an increase of $|E_z|$;

(b) The electric fields due to the 'Wilson monopole' are screened to an increasing degree in their turn, as time progresses, by the capacitive spatial charges of opposite polarity, so that $|E_z|$ decreases in this period.

2. Maximal QESF intensity decreases with altitude up to about 80 km much slower than the breakdown electric field. As a result, QESF due to strong enough lightning discharges become possible to cause conventional breakdowns in the mesosphere and the upper stratosphere.

3. QESF reaches its maximum at a time moment within the interval limited by the characteristic lightning discharge time and the local relaxation time.

4. QESF maximum persists at altitudes with $\tau_L < \tau_R$ for the time period [τ_L , τ_R].

5. For occuring a sprite at night, a lightning with a charge moment change bigger than 1000 C×km is necessary.

The obtained results are approximate, since QESF is not considered in self-consistent dependence on the conductivity modifications due to electron heating. Also, the anisotropy conductivity ion the ionosphere is neglected. The anisotropy may modify the distribution of QSF in the lower ionosphere, especially at equatorial latitudes, where the electric fields spatial distribution is very sensitive to [11, 12]. More accurate analysis needs model development in further works.

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