The Solar Rotational Activity Variations during the 23-th Solar Cycle

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The study of the solar activity variability has been of great interest since its discovery. On the one hand it is important for the understanding of the Sun as an active star and on the other hand – for the investigations of the solar-terrestrial connections. The solar magnetic field reverses approximately every 22 years, and manifests the 11-year solar cycle, in which the Sun changes its activity from its maximum value to the minimum one. The activity variations, developed by the sun surface rotation in connection with the nonsymmetrical distribution of active regions over the solar disc appear in a shorter time scale. As it is well known, these variations have periods of about 27 days. The solar surface rotates with different velocity, depending on the latitude. The differential solar rotation period, observed from the Earth, varies from 26.75 days at the solar equator up to approximately 29 days at higher latitudes. However the observed periodicity is generally in a wider range: from 20 up to 36 days. This wider spread is a result of the combination of both active-region evolution and solar rotation. A simple empirical solar activity model is proposed, which describes the obtained behavior by harmonic oscillations with simultaneous amplitude and phase modulation. The solar rotational periodicity is analyzed using wavelet. It is demonstrated, that the model describes well the separate episodes of the active region evolution. Both kinds of modulations are the consequence of activity region growth or decay and hence, they are a result of a variable pattern of spots and active regions on the solar surface.

Introduction

The solar ultraviolet (UV) irradiance between 120 *nm* and 300 *nm* controls the radiative, chemical and dynamical processes in the middle and upper Earth's atmosphere, by absorption, photodissociation and radiative heating processes of the atmospheric oxygen molecules O_2 and the ozone. The the study of the solar variability is also of interest for the understanding of the Sun as an active star on the other side for investigation of solar-terrestrial connections.

The solar activity varies by the Sun's surface rotation in connection with the nonsymmetrical distribution of active regions over the solar disc. It is well known that these variations have periods of about 27 days. The amplitude of the solar irradiance variations strongly depends on its wavelength and increases towards shorter wavelengths. Over a solar cycle, the total solar irradiance changes with only 0.1%, yet in the UV and shorter wavelengths they exceed those by orders of magnitude [1]. The changes in this spectral range are of basic importance for the solar-terrestrial influences and require detailed analysis of the growth and decay of active regions, because the extreme UV (EUV) solar irradiance is fully absorbed by the Earth atmosphere, and the UV irradiation is absorbed partially.

27-day Solar Cycle

In the beginning of the solar cycle, the active regions originate at heliographic mid latitudes Φ and shift equatorwards with the cycle extension. The solar surface rotates with different velocity, depending on the latitude. The differential solar rotation period, observed from the Earth, can be approximated with sufficient accuracy by: $P_{syn} = 26.75 + 5.7 \sin 2\Phi$ [2]. A wider spread of the solar activity indices is observed, however, around the 27-days period. At first, the 27 periodicity is non-sinusoidal and higher harmonics can appear in the spectra. Donnelly and Puga have shown that the second harmonic (the 13.5 day oscillation) is of physical nature and includes predominantly two groups of activity regions with a phase shift of approximately 180° [3]. A 12-14 day period is obvious in most indices, related to chromospheric emissions, such as the CaII-K plage index, but is absent in the F10.7. However, the periodicities generally range from 20 to 36 days. It is pointed out that the indices with "periodicities around 30-34 days shifted to about 26-28 days in 1982", when the Sun was in a maximum activity phase. Bouwer concludes that the "period is a result of the combination of active-region evolution and solar rotation (i.e. not due to the differential rotation) [4]. A reason for the widely spread periodicities around 27 days is that the dominant radiance within an active region can move rapidly forwards or backward in respect to the solar surface [5]. The power spectra of projected areas of active and passive spots show the 27-day rotation, active spots besides 28 and 25, 51 and 157 day periodicities [6]. Bai [7] remarks that the periodicity at approximately 25 and 50 days are maybe harmonics of the Rieger's period (about 150 days). The power spectra of projected areas of active and passive spots show the 27-day rotation, active spots except the 28 and 25, 51 and 157 days' periodicities [6].

Are the 27-day activity variations exact to be analyzed by Fourier transform?

The solar activity periodicity changes systematically due to the differential rotation and the characteristic sunspot development during a solar cycle (butterfly diagram). This means that the solar activity periodicity depends on the phase (increasing, decreasing, maximum or minimum) of the solar cycle. The activity varies by growth and decay of active regions and by their possible to move fast over the solar surface. When an active region exists and a new region is developing, the activity center of the whole solar disc changes its position, the consequence of which is the change of the periodicity of the solar activity indices, referring to the whole solar disc. Besides, the combination of the active-region evolution and the solar rotation causes a change of the 27-day periodicity. It should be concluded that the solar activity variability at the time scale of the 27 days is not a strong periodic process and the Fourier transformation is not suitable for the periodicity analysis of the solar rotational activity variability. To study the time dependencies from a periodical or quasi-periodical signal, a method, often used, is to sub-divide the time series and analyze each sub-sample separately through a periodogram analysis. Another method is to analyze sub-samples by window propagating over the entire series and collecting the Fourier transforms (Gabor transform). The disadvantage of the Gabor transform is that the frequency and time space resolution a priori is limited by the window function application. That is why to study the evolution of the solar activity periodicities and the atmospheric response to the solar activity variability, the wavelet analysis is used in this paper. In contrast to the Fourier spectrum, the wavelet spectrum gives both information on the periodicities and information at what time these periods occur.

Data set preparation

For our analysis we have used the CaII-K-line index from the Big Bear Solar Observatory (BBSO) for the time period from 1996 until 2003 (http://www.bbso.njit.edu/). The index is determined by solar images in the K-line (393.9 nm) with a 1.5Å FWHM filter, obtained by a CCD camera. The pixel numbers above the threshold of 7.5% produce the plage area index (D75A - index 05 in the BBSO data archive), which shows strong rotational modulation. The year 1996 is approximately the solar activity minimum and in October the current solar cycle 23 begins. To represent the 27-day rotational modulation, the time series are separated in four parts.

In the data set the only missing data are for successive days over a time interval of some days and very rarely for a longer period. The missing data are linearly interpolated (with the values at the interval end points) and for intervals longer than 5 days, the corresponding day number is stored. Smoothed by running mean over 41 days series were subtracted from the interpolated time series to bring out the periods about 27 days. For data gaps longer than 5 days, the values of the time difference series are set to zero, because the interpolated values over larger time intervals differ strongly from the real values. The rotational amplitudes are very different during the separate activity phases. They are very small at the solar minimum and then they continuously increase with the activity increase. The changes of the activity indices by the Sun rotation are much more regular around the solar maximum.

The subtraction of the smoothed time series is not essential for the application of the wavelet analysis, and the damping of the low frequency parts has only a visualization effect. As a result of this filtration, however, the wavelet spectrum shows only the interesting higher frequency spectral part. To investigate the duration of separate the solar rotations, observed in the CaII-K line, we have used the wavelet analysis.

Wavelet basics

The wavelet analysis is successfully applied in the last years for analysis of geophysical time series [8, 9]. Hempelmann has developed a method to determine the duration of the Sun differential rotation [10] and for stars, rotating as slow as the Sun [11]. Soon and Baliunas have found by a wavelet time frequency approach that the rotation of some chromospheric active stars is at least 10 years long [12]. The long-period solar activity wavelet analysis was carried out by Le [13]. A wavelet is a function and satisfies the admissibility condition:

$$C_\psi = \int_{-\infty}^{\infty} \left|\hat{\psi}(\omega)\right|^2 \frac{d\omega}{|\omega|} < \infty \qquad (1)$$

and is normalized

$$\|\psi\| = 1 \ . \qquad (2)$$

The condition (1) under certain circumstances is equivalent to the condition of zero mean:

$$\int_{-\infty}^{\infty} \psi(t)\, dt = 0 \qquad (3)$$

The continuous wavelet transformation (WT) $W\psi$ for a given mother wavelet of a time signal $f(t)$, is defined as:

$$W_\psi f(a,b) = \int_{-\infty}^{\infty} f(t)\psi^*_{a,b}(t)\, dt \qquad (4)$$

with the complex conjugate wavelet function

$$\psi_{a,b} = \frac{1}{a^{1/2}}\psi\left(\frac{t-b}{a}\right), \qquad (5)$$

where $\psi_{a,b}$ represents the mother wavelet ψ, shifted by b and dilated/compressed by a. Factor $1/a^{1/2}$ satisfies the normalization condition. Since the beginning of the 1980s, when the wavelet theory was first developed, many mother wavelets have been found. The simplest of them are the Haar- and Morlet-wavelets and the Mexican hat. The function

$$\psi = \left[1-(ct)^2\right]e^{-(ct)^2/2}, \qquad (6)$$

is the so-called Mexican hat wavelet, because the graph of this function is similar to it [14]. The Mexican hat is a simple real function and the calculation of WT is comparatively fast. Moreover, the function is symmetric, which is useful for identifying regions of maximum/minimum curvature [15].

The mother wavelet developed by Morlet is

$$\begin{aligned}\psi(\mathrm{t}) &= \pi^{-1/4}\exp(\mathrm{i}\omega_0 \mathrm{t})\exp(-\mathrm{t}^2/2)\\ \omega_0 &> 5\end{aligned} \qquad (7)$$

In analogy to FT, the spectrum is given by $\left|W_\psi f\right|$ and the power spectrum by $\left|W_\psi f\right|^2$. The result of the WT $W\psi(a,b)$ is defined in the phase plane, spanned by the time shift b and the scaling parameter a. For the chosen wavelet with $c = 4$ the scale parameter a at maximum and minimum of $W\psi(a,b)$ presents directly the signal period. For the interpretation of the wavelet spectrum it is important to note that the frequency resolution for constant b is proportional to $1/a$ and it decreases with the increase of ω, but the time resolution increases with the decrease of a. This is the typical wavelet zoom effect. In consideration of the frequency filtering of a given time series it should be noted that WT is similar to a convolution (For $a = 1$ the WT is exactly the convolution of the time signal $f(t)$ with the analyzed wavelet.) The time series are filtered by a more stretched /compressed filter function (mother wavelet) by increasing/decreasing scale parameter a. A practical problem for the wavelet application is the determination of the equivalent Fourier-period T to the scale parameter a, for which a period is observed at the wavelet spectrum. Table 1 show the analytic formula to calculate the equivalent Fourier-period.

TABLE 1
Scale parameter a and its connection the e-folding time and the equivalent Fourier-period

	e-folding time	Fourier-period T	for T = a
Mexican hat	$\sqrt{2}a$	$\frac{2\pi}{\sqrt{5/2}}a$	3.97
Morlet	$\sqrt{2}a$	$\frac{4\pi}{\omega_0+\sqrt{2+\omega_0^2}}a$	6.20

Simple solar activity model

The CaII-K line time series shows, that the short-time periods are modulated by periods of several months (see also Fig 5). The obtained behavior can be described in first approximation by a simple model, which consists of a harmonic oscillation with simultaneous amplitude and phase modulation:

$$B = A\cos[[t - t_1)\omega], \quad (8)$$

where A is the amplitude modulation:

$$A = 1 + \cos[(t - t_2)\,\omega_l] \quad (9)$$

and ω is the phase modulation

$$\omega = \omega_0\{1 + d * \cos[(t - t_3)\omega_2]\} \quad (10)$$

and t_1, t_2, t_3 are the phase shifts. The Period of the Sun rotation is T_0 and it is chosen to 27-days and ω_0 is the corresponding angular frequency. The period of the amplitude modulation ω_l is selected to be several Sun rotations. We have fixed ω_l to $n\omega_0$, with n=8 (n is not required to be an integer number). We have set the phase shifts t_1 to $T_0/2$ and t_2 to $nT_0/2$. In that way the amplitude maximum of A is reached at half of the period length. We have studied only the influence of the phase shift t_3. The modulation index d is fixed at 0.1. For a further simplification $\omega_l = \omega_2$ is set. Thus, the phase modulation describes the growth or decay of the period during the active region evolution expressed by amplitude modulation.

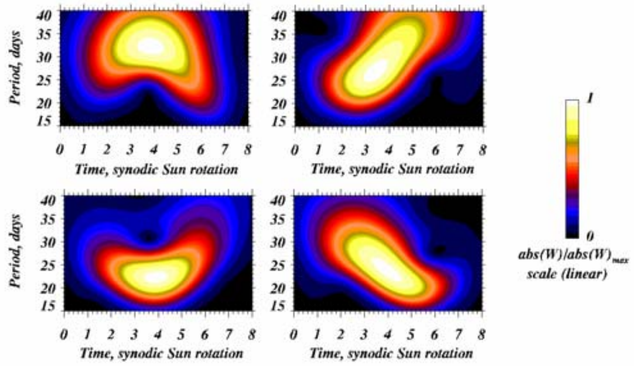

Fig. 1. Wavelet spectra of the modeled solar activity of the first period for some phase shifts t3. Top left: t3=T0, the maximum of the amplitude of solar rotation activity variation coincides with the maximum of the amplitude modulation located at 4 Sun rotations, right: t3=3T0, the period of solar activity slowly increased, however the maximum of its amplitude reached before the period maximum and the activity decreases suddenly. Bottom left: t3=5T0 the maximum of the solar activity variation coincides with the minimum of the reached period, and right: t3=7T0, the solar activity in the beginning of the signal is relatively low but the period is high, with increasing the time the period decreases and after the transition of the activity maximum it decays.

Results

The time series and the calculated wavelet spectra are shown in Fig. 2. For better color adjustment the time series from 1996 to 2004 was calculated separately for 2 years and the last series from 2004 to May 2005. The time series display obvious wave packets with period lengths of several months, characterizing amplitude modulation of the rotational solar activity. In Fig. 2 several clear examples are marked in the at the time series by a straight line under the packet. These periods are probably related to active region growth and decay, i.e. the typical lifetime of a single spot group or an active region. The area and the brightness of an active region decay/grow after the plage formation. The parent spot of a bipolar group of a developing region travels slightly forward with increasing the distance to the subsequent spot. With decrease of the active region, the parent spot shifts backwards nearly to its initial heliographic longitude, the bright remnants always trailing behind and towards the poles. The appearance or the decay of new regions changes the activity distribution over the solar disc. These processes modify the observed rotation period.

At the long time from December 1999 to September 2000 the time series show not clear modulated packets. Subsequently the wavelet for this time span show low periodicity changes of the solar activity.

As one can see from the calculated wavelet spectra the periods change from 22 up to 34 days, which is significantly longer than the predicted only by the differential solar rotation.

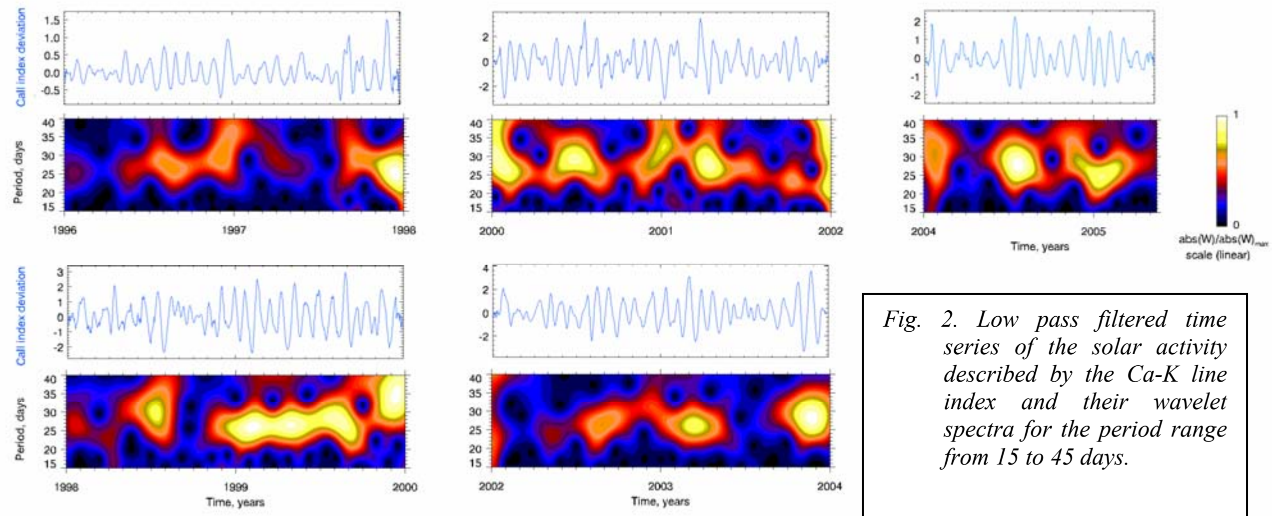

Fig. 2. Low pass filtered time series of the solar activity described by the Ca-K line index and their wavelet spectra for the period range from 15 to 45 days.

Conclusions

The simple solar activity model, which takes into account phase and amplitude modulations, can be used to explain that the periods due to the solar 27-rotation cycle are in a range of 22 up to 34 days and both kinds of modulation are the consequence of activity region growth or decay, hence, they are a result of a variable pattern of spots and active regions on the solar surface. The areas and the brightness of an active region grow after the plage formation. The parent spot of a bipolar group of a developing region travels slightly forwards with increasing the distance to the next spot. With decrease of the active region, the parent spot shifts backwards nearly to its initial heliographic longitude, the bright remnants always trailing behind and towards the poles. The appearance or the decay of new regions changes the activity distribution over the solar disc. These processes modify the observed rotation period. The wavelet analysis clearly shows a change of the period from 22 up to 34 days, which is significantly longer than that, predicted by the differential sun rotation only. We have described the variability of the longer period by amplitude modulation, while the shortening and the prolongation of the solar rotation period can be modeled by a phase modulation. The superimposing modulations change the period more than twice by nonlinear effects. The proposed simple empirical model describes with good quality the separate episodes of the active region evolution. Both kinds of modulation are the consequence of activity region growth or decay and hence, they are a result of a variable pattern of spots and active regions on the solar surface.

The applied method based on wavelet analysis is very helpful to study of the duration of several solar periods and their progress.

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