Magnetic field configuration in corona and X-ray sources for the flare from May 27, 2003 at 02:53

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Abstract. Numerical MHD simulation above the real active region of the solar corona shows the current sheet creation before a flare. The system of graphic output is developed here, which permits to study the magnetic field configuration obtained from the presented MHD simulation. The physical meaning of the flare energy accumulation and rapid release processes is best demonstrated by the lines in the plane of the current sheet configuration which are tangential to the projections of the magnetic field vectors on this plane. The magnetic j×B/c forces are perpendicular to these lines. Position of such lines defines the magnetic forces, which create the current sheet and then destroy it, when the system transforms in unstable state. The magnetic lines can be analyzed in complicated magnetic configuration near a current sheet and in corona above the active region using the developed graphical system.

Keywords: sun, solar flare, current sheet, magnetic field configuration

Introduction. MHD simulation of the solar corona in order to define the solar flare mechanism.

Primordial energy release of the solar flare takes place in the solar corona at the altitudes 15 000 - 30 000 km, as proved by high resolution observations of thermal X-ray emission from flares on the limb (Lin et al., 2003). The flares take place above the active regions on the photosphere with magnetic field about 3000 G. The typical background magnetic field on the photosphere is ~ 1 G. The flares are observed above the active regions when the magnetic flux on the photosphere reaches ~ 10^{22} Mx (Ishkov, 2001; Podgorny and Podgorny, 2013c; Podgorny et al., 2015). The magnetic energy of a flare is stored in the solar corona, where the magnetic field configuration cannot be defined from the observations. It is necessary to find the magnetic field configuration and plasma parameters in the corona above an active region by numerical solving of magnetohydrodynamical (MHD) equations using the observed magnetic field distribution on the photosphere for setting boundary conditions. It is difficult to define magnetic field in the corona in this way because of numerical instabilities which appear in first turn near the photospheric boundary where there is the strong magnetic field gradient. Another difficulty is to find of complicated magnetic field configuration in 3D space even if the magnetic field vector is known with sufficient precision in each point of the space. Here we present the attempt to study the magnetic field configuration in the position of flare energy release and in the all region of the corona under the consideration for existing results of MHD simulation using the specially developed methods.

It has been shown that the current sheet position coincides with the position of thermal X-ray emission source which appears during the solar flare (Podgorny and Podgorny, 2013a; Podgorny and Podgorny, 2013b). The MHD simulation in the solar corona above the active region NOAA 10365 shows appearance of the current sheet during the flare May 27, 2003, which began at 02:40 UT. The comparison is carried out at the X-ray maximum intensity 02:53 UT. Such coincidence can be considered as independent confirmation of the flare mechanism, according to which an explosive release of the current sheet magnetic energy is occurred in the solar corona. Magnetic field dissipation causes heating of the plasma, and therefore appearance of the thermal X-ray emission source at the site of the flare energy release. The current sheet mechanism explains the accumulation of flare energy in the corona as a result of magnetic field slow evolution on the photosphere, and then its fast release. The current sheet is created in the vicinity of a magnetic field X-type singular line due to magnetic field deformation by the plasma flow which is caused by **j×B**/c force. During quasi-stationary evolution of the current sheet the plasma density near it decreases in time (see for example Podgorny, Podgorny, 2003; Podgorny and Podgorny, 2012a). Then the current sheet transforms into an unstable state, and explosive release of its magnetic energy takes place (Podgorny, 1989).

Basing on the mechanism of energy release in the current sheet and using the results of numerical simulation the solar flare electrodynamical model is developed which explains its main observational manifestations (Podgorny and Podgorny, 2006; Podgorny and Podgorny, 2012b). The Induction electric field that caused by the rapid change of the magnetic field during the instability accelerates protons along a singular line up to the energy ~ 20 GeV, that leads to the appearance of solar cosmic rays during some flares (Balabin et al., 2005; Podgorny et al., 2010). Plasma acceleration upward along the current sheet causes the coronal mass ejection. The sources of hard X-rays emission are located at intersection of the magnetic lines coming out of current sheet with the solar surface. The X-ray emission is a result of the interaction of electrons accelerated in the field-aligned currents with lower dense layers of the solar atmosphere. The field-aligned currents are created in the current sheet as a result of Hall effect due to electron acceleration along the current sheet by jB_n force (product of current density in the current sheet on the magnetic field component which is perpendicular to the sheet).

For precise study of the flare situation at setting of conditions for MHD simulation no assumption about the solar flare mechanism has been done. All conditions for MHD simulation are taken from observations. The magnetic field measured on the photosphere by SOHO MDI (http://soi.stanford.edu/magnetic/index5.html) is used for setting boundary conditions. Conditions for setting other values on the photospheric and nonphotospheric boundaries are approximated by free-exit conditions. The solution is started several days before the flare when there are no strong disturbances in the corona, and so the potential magnetic field calculated using magnetic field distribution on the photosphere is used for setting the initial condition. The conditions of simulation are described in detail in Podgorny and Podgorny (2008). The initial potential field does not contain free magnetic energy, therefore no flare mechanism is assumed. In other works (see, Lin, 2004; Amari et al., 2003; Kusano et al., 2003) and works considered in the review (Podgorny and Podgorny, 2012a)) initial conditions were set in such a way that it is assumed the mechanism of solar flare. The initial magnetic field contains stored flare magnetic energy in structures such as magnetic ropes and/or configurations with magnetic helicity.

The finite-difference scheme which is stable for large steps is developed by authors (Podgorny and Podgorny, 2004) to accelerate calculation. The scheme was realized in the PERESVET code. This scheme is absolutely implicit, and it is conservative relative to magnetic flux. This scheme stabilizes a slowly developing instability, which appears near the photospheric boundary where the field gradient is high. The magnetic flux through the grid cell boundary, divided by the boundary square, is used in such a scheme instead of the magnetic field vector component. A high stability level of the scheme was maintained because the dissipation and transfer terms were taken at the next time step, and a first order upwind approximation of the transfer terms was performed in this case. The scheme was solved using the iteration method. The iteration convergence was good because the values at the central stencil point, included in the finite difference analogs in the right-hand side of the equations, were taken during the next iteration. Application of this method is the main distinction of our scheme from other finite-difference schemes used for MHD simulation in the solar corona. In other published works finite-difference schemes are explicit. Some of them (DeVore, 1991), use the Riemann wave method with upwind approximation of the Riemann invariant transport terms, and then the correction (FCT or TVD) is performed by addition of the anti-diffusion term to obtain high order approximation. This correction has been used only in some parts of the computational domain, where the solution is sufficiently smooth, and there are no discontinuities. Otherwise the stability of a finite-difference scheme is violated. Other finite-difference schemes (Arber et al., 2001) employ the Lagrangian grid with further recalculation to the Eulerian grid, using of special interpolation procedure, and employ the magnetic viscosity term to enhance scheme stability. These schemes are more precise than ours in the places of smooth solutions (the precision is violated on discontinuities in each scheme). Our implicit scheme can simulate details of disturbance propagation in the corona and plasma behavior during current sheet creation. Possibly, the more precise scheme can simulate, the better some details of wave interactions, but our paper is not supposed to perform such analysis. The main our purpose is to perform simulation as fast as possible.

In spite of specially developed methods using, MHD simulation in the corona is carried out so slow that it is impossible to perform calculation for active region NOAA 10365 on common personal computer. Therefore calculations were performed in the time scale, which was reduced by 10^4 times. It means that magnetic field evolution on the photosphere, which is performed during a day in real active region, is performed during ~ 10 seconds in simulations.

For such a fast changing of the magnetic field the instability is developed near the photospheric boundary. The instability distorts the solution. However the methods which have been used permit to stop of instability propagation inside the computational domain from the photospheric boundary. The methods also permit to restrict instability growth near the photospheric boundary so that calculation can be performed. The graphical methods are developed that can be used for calculation in the real time scale, which can be performed using supercomputer. Results of calculation show that these methods are useful for understanding the physical processes in the corona using the obtained results of MHD simulation, because the coincidence of calculated current sheet position and the thermal X-ray source position implies the sufficient precision of calculation at least near the flare site.

The search methods of magnetic field configuration that obtained in solar corona by numerical MHD simulation

A graphics system is developed to search for the flare position in the results of the MHD simulation and visualization of MHD equations numerical solutions in the solar corona. The method of search of flare positions uses that property of the sheet, according to which the local maximum of the absolute value of the current density is located in the center of a current sheet regardless of the coordinate system. All local maxima of the current density should be found, and then near each maximum the magnetic field configuration is analyzed. The detail description of the flare position search system is contained in Podgorny and Podgorny (2013b). The usage of this search system permits to find the position of flare May 27, 2003 at 02:53. It coincides with the position of thermal X-ray emission source.

To confirm the solar flare mechanism, it is necessary to get the magnetic field configuration in place of the flare itself, and in a sufficiently large region of space. To study the explosive process in plasma of the solar corona the magnetic field configuration near the current sheet must be known. According to the solar flare electrodynamical model the nonthermal hard X-ray sources appear in the places of the photosphere crossing with the magnetic lines, which are going out of the current sheet. So to understand the observational manifestations of the flares it is necessary to know magnetic field configuration in the large area of solar corona (its size is ~ 100 000 – 300 000 km) above the active region, where the flare occurs. After the recent modernization of the graphical system, the possibility appears to study conveniently the magnetic field configuration.

Magnetic field configuration during the flare in the corona and near the site of flare energy release.

MHD simulation is performed in the solar corona in a computational domain which is rectangular parallelepiped. Its lower boundary is located on the photosphere. It includes the active region. It is a square, whose 400 000 km length is taken as a dimensionless length unit. The domain size is ~ 4 times greater than the linear dimension of the active region. The center of this square is located in the middle of the active region with coordinates (118.3", -87.27") on the solar disk at 2:53 UT on 27 May 2003. The XZ plane (y = 0) is on the photosphere, with X axis directed westward and the Z axis directed southward. The Y axis is directed outward from the Sun and normal to the photosphere, the size of the computational domain along Y axis is 120 000 km (0.3 in dimensionless units). The origin of coordinates is located in the north-east corner of the computational domain. Thus, the computational domain is rectangular parallelepiped of the form (0<x<1, 0<y<0.3, 0<z<1). The problem solving for simulation in the active region NOAA 10365 and the computational domain are described in detail in Podgorny and Podgorny (2008).

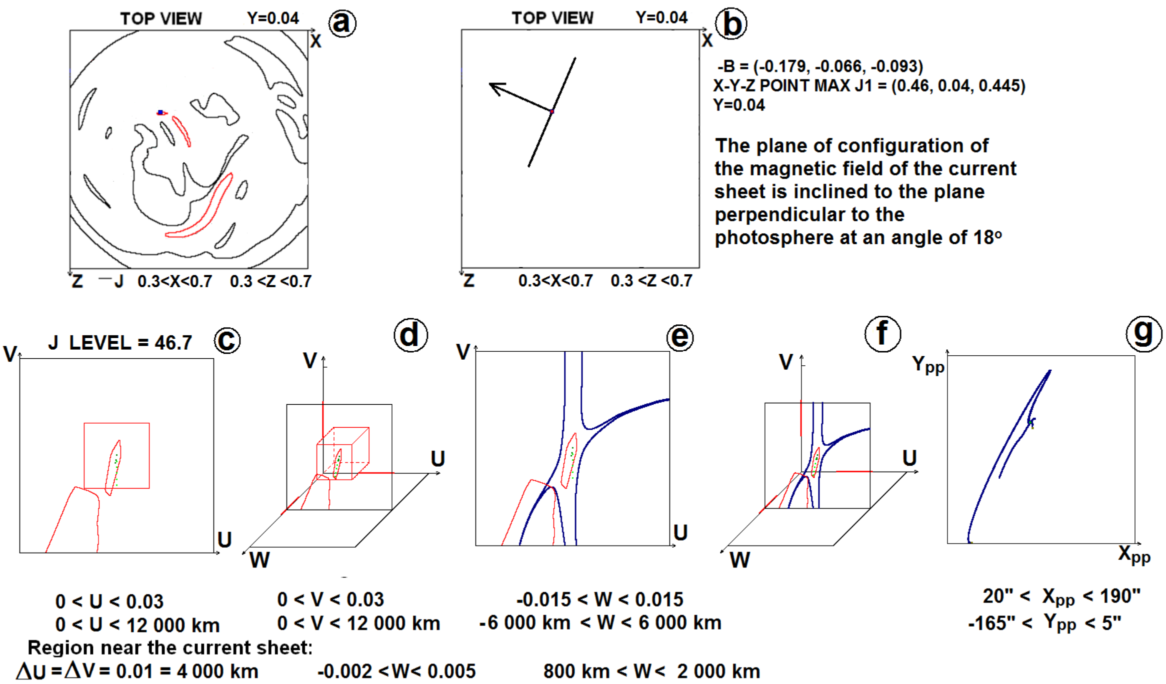

Fig. 1. Current sheet position and lines of equal current density in the plane y=0.04 which is parallel to the photosphere and passing through the center of the current sheet (a). Projection of magnetic field vector in the center of the current sheet on the plane y=0.04 and the line of intersection with the plane of the current sheet configuration with the plane y=0.04 (b). Region of current sheet on plane and in space, lines, which are tangential to projections of magnetic vectors on the plane of current sheet configuration (c-f). Projection of these lines on the picture plane (g).

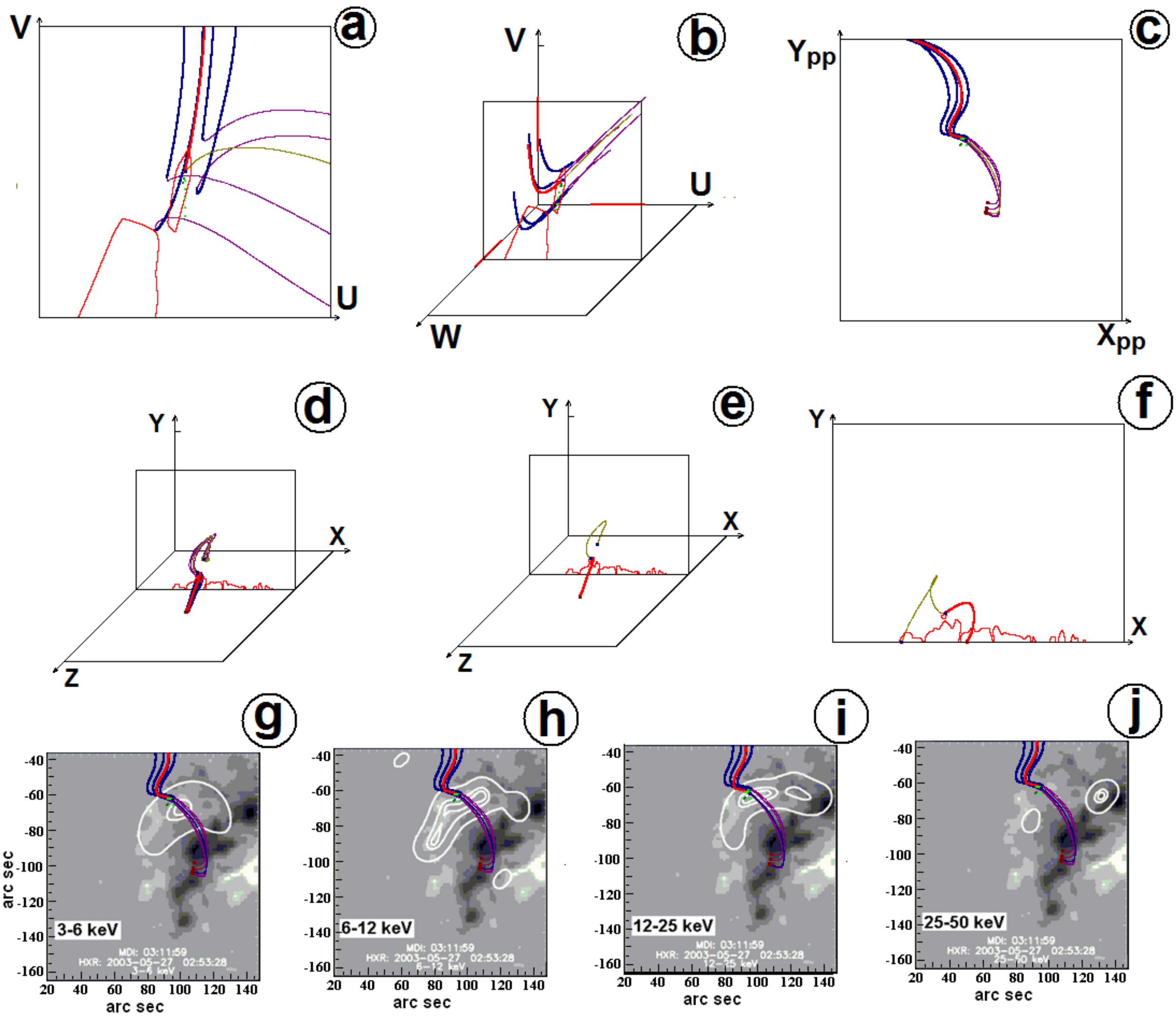

Fig. 2. Projections of magnetic lines on the plane of current sheet configuration (a), magnetic lines in 3D space near the current sheet (b), their projections on the picture plane (c). Magnetic lines in corona (d, e), projection of magnetic line, which pass through current sheet center, on the plane z=0.445 which is perpendicular to the photosphere (f). Projections of magnetic lines on the maps of X-ray emission intensity distribution (g-j).

The current density maximum in the current sheet for the flare May 27 2003 02:53 is located in the point (0.46, 0.04, 0.445) (Podgorny and Podgorny, 2013a; Podgorny and Podgorny, 2013b). Its position on the plane y = 0.04 parallel to the photosphere is marked by a blue square sign in Fig. 1a. Also lines of the constant current density are drawn on this plane. Magnetic field configuration of the current sheet is the most distinctly pronounced in the plane, which is perpendicular to the magnetic field vector in the center of the current sheet (point of current density maximum). In this plane the magnetic field with oppositely directed lines, obtained by deformation of X-type magnetic configuration, by the best way corresponds to sheet current density distribution. We call this plane the plane of the current sheet configuration. The magnetic field vector in the point (0.46, 0.04, 0.445) is **B** = (-0.179, -0.066, -0.093) in dimensionless units or **B** = (-53.7, -19.8, -27.9) in Gausses (300 G is taken as dimensionless unit).

The current sheet orientation can be seen in Fig1b. The projection of the magnetic field vector in the center of the current sheet and the line of intersection of the current sheet configuration plane with the plane parallel to the photosphere, which pass through the point of maximum (0.46, 0.04, 0.445), are shown. Magnetic field component along the y-axis is negative (B_y = -0.066, or B_y = -19.8 in Gausses), therefore, the plane of the current sheet configuration is tilted to the plane of the photosphere at the angle 18^0, so that the top part of this plane is rejected along the projection of vector. To study the field configuration in the vicinity of the current sheet, the image in Fig. 1c-f displayed on the plane in the square or in the volume of a cube with the side of 0.03, the centers of which are located in the center of the maximum point (0.46, 0.04, 0.445). W axis is directed along the magnetic field vector **B** = (-53.7, -19.8, -27.9). U and V axes are located in the plane of the current sheet configuration, so that the U-axis is directed along the line of intersection of the current sheet configuration plane and of the plane containing the magnetic field vector **B** = (-53.7, -19.8, -27.9) and the Y-axis. Figure 1g displays the picture plane in the area (20" <x <190", -165" <y <5") (its linear size is 170" or ~120 000 km or ~0.3 of dimensionless unit) of the solar disk, for which there are maps of soft and hard X-ray emission for the flare May 27, 2003 at 02:53 obtained by the spacecraft RHESSI (http://rhessidatacenter.ssl.berkeley.edu) which are superposed on the magnetogram.

The current sheet is situated in the region $\Delta u=\Delta v=0.01$, 0.005<w<0.002 (in dimensionless units, the length unit is 400 000 km), its boundaries are shown by the red thin line in Fig. 1c, d. The maximal value of current density in the plane perpendicular to the W-axis is changed not more then on 10%, and outside of these limits (w<-0.005, w>0.002) it strongly drops. Therefore, to study of the magnetic field configuration, besides the lines passed through the plane of the current sheet configuration w=0, it is also selected lines passed through the planes w=-0.005 and w=0.002 (see. Fig. 3 below). Figures 1e and 1f displays the lines tangential to the projections of the magnetic field vectors on the plane of the current sheet configuration, passing through the points situated at a distance of ~ 0.001 from the center of the current sheet. The positions of such lines permit to understand the physical meaning of current sheet creation, since it shows the direction of magnetic forces perpendicular to these lines which causes such a motion of plasma, that the plasma inflows to the sheet in direction perpendicular to it and flows out along the sheet. This plasma flow must deform the magnetic field into configuration of the current sheet. The forces perpendicular to lines which are tangential to the projections of the magnetic vectors on the plane of the sheet configuration create the current sheet with the energy accumulated in its magnetic field, and then destroy the sheet when the system transforms in an unstable state. These forces arise due to appearance of the current along the singular line which is perpendicular to the plane of the current sheet configuration. The normal to this plane magnetic field component does not influence on the magnetic forces because it is directed along the current. Therefore the forces are defined only by field components in the plane i.e. by projections of magnetic field vectors on the plane, tangential of which are shown in Figures 1e and 1f. The projections of these lines, which are located completely in the same plane, on the picture plane are shown in Fig. 1g.

Fig. 2a-c presents the magnetic lines passing near the center of the current sheet. Four lines pass through the same points in the plane of configuration as the points of the tangential to vectors projections in Fig. 1e and 1f. Parts of them are shown as bold blue lines in front of the configuration plane, and the parts of these four lines behind this plane are shown as thin violet ones. Another line passes through the center of the current sheet; it is shown as a bold red line in front of the configuration plane. This line is shown behind this plane as a thin brown one. Taking into account all the magnetic field components of the magnetic line makes the configuration of the field so complicated that it is difficult to understand the directions of magnetic forces if we know locations of magnetic lines.

Using the method of analysis of the lines tangential to the projection of the magnetic vectors in the plane of the current sheet configuration, it is possible to determine whether the maximum of current density is the current sheet, or it is just an occasional maximum. In the case of current sheet the configuration of lines tangential to the magnetic vectors projections on the plane of current sheet configuration must present X-type configuration. It is sufficient to use only this method to determine the current sheet. Probably it is the most convenient method because it does not demand to find distributions of current density, plasma density and temperature, which can be determined only approximately if calculations are performed with low precision as in our case. When we will perform more precise simulation it will be possible to use additional methods which are based on the property of increase of current density, plasma density and/or temperature in the current sheet. Some other methods are used by Sui and Holman (2003) to prove from observations the formation of a current sheet in the flare April 15, 2003, which took place on the west limb. The high temperature (~30 MK) in the place of flare obtained from analyze of the thermal X-ray emission in the range 3-25 keV and considerations about some features of magnetic field configuration on the base of X-ray emission source shape in the picture plane permit to make conclusion about the formation of current sheet in the solar flare. The result of Podgorny and Podgorny (2013a) about the coincidence of positions of calculated current sheet and observed thermal X-ray emission source, and presented here result about magnetic field configuration of this current sheet, confirm conclusion about the formation of current sheet in the solar flare. It is very possible, that appearance of thermal X-ray emission source during the flare April 15, 2002 means the existence of the current sheet. However, there is a small probability that the longitudinal magnetic field component is absent and all magnetic lines are situated in the same plane. For such classical current sheet field configuration all the magnetic sources on the photosphere with alternated polarities must be situated on the same line. Such situation is practically never realized. It is very possible, that magnetic field in the current sheet in the flare April 15, 2002 have the longitudinal component and its configuration is as complicated as one considered here for the flare May 27, 2003. Probably

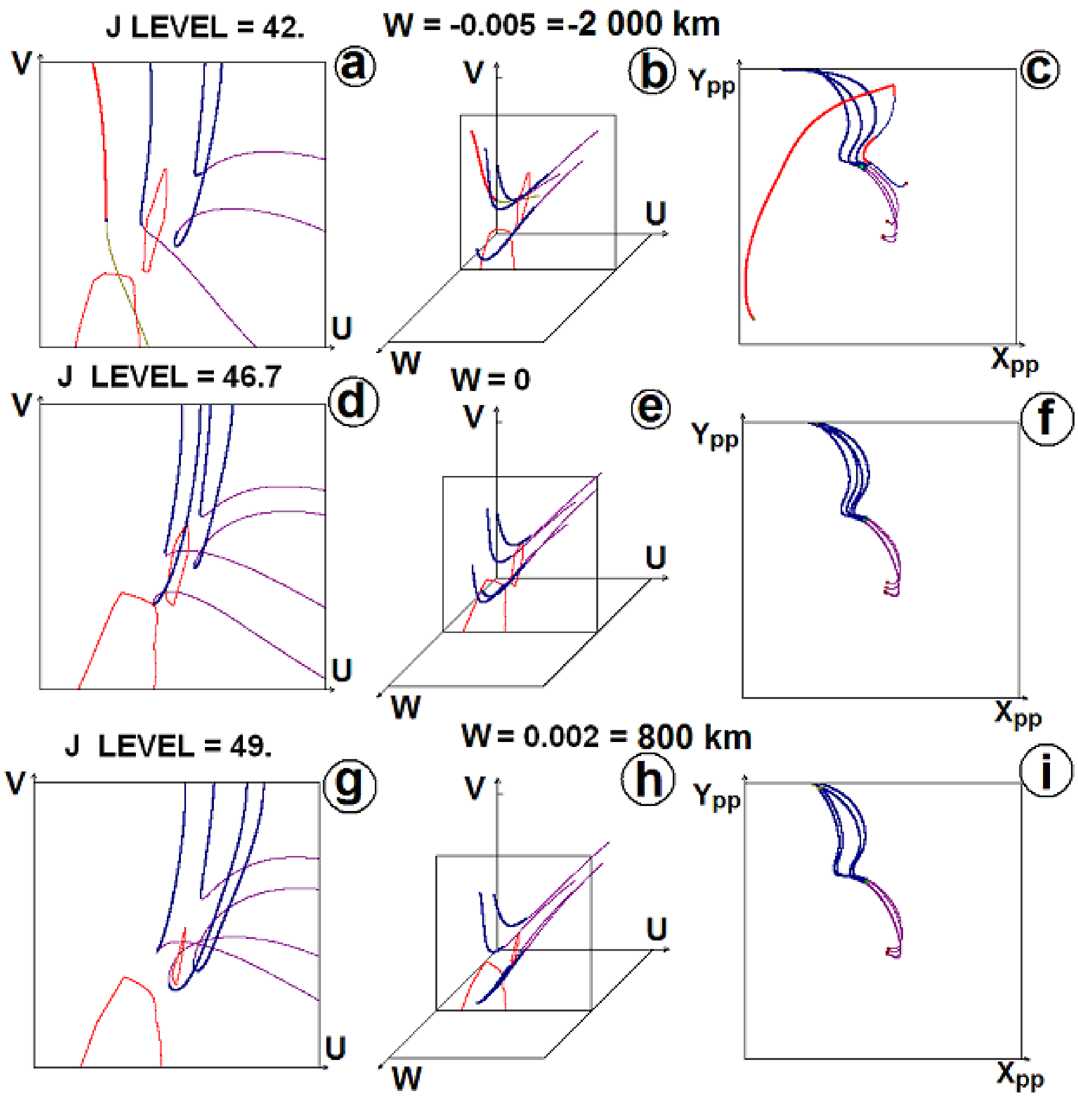

Fig. 3. Magnetic lines passing through the points located near the current sheet center in the planes w=-0.05 (a, b, c), w=0 (d, e, f) and w=-0.05 (g, h, i). Projections of these lines on the plane of current sheet configuration (a, d, g), their arrangement in space (b, e, h), and their projections on the picture plane (c, f, i).

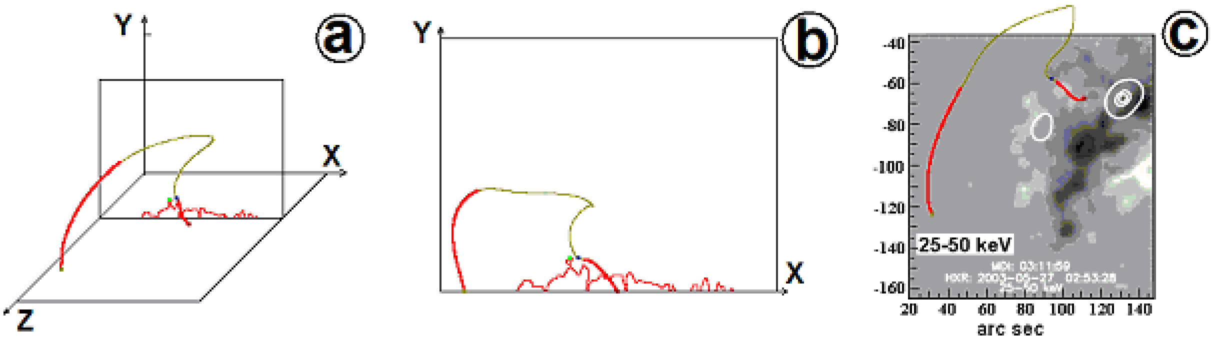

Fig. 4. The line which passes through the point near the current sheet center and which intersects the photosphere near the source of hard X-ray emission. Intersection of this line with the plane z=0.445 is marked by blue point. The thin red line on panels (a) and (b) is the line of the constant current density and center of current sheet is marked by green point in the plane z=0.445.

these problems can be solved only if we know magnetic field configuration near the site of flare energy release with sufficiently high precision and the methods developed here can help solve such a problem.

Fig. 2d presents these lines (which are shown in Fig. 2a-c) above the active region in the computational domain situated in the corona. The plane z=0.445 is situated perpendicular to the photosphere and it contains the center of current sheet. The line of intersection of this plane with the photosphere is parallel to the solar equator. It can be seen that the location of the selected magnetic lines are not very different from the location of the line passing through the center of the current sheet (Fig. 2e). The projection of the line passing through the center of the current sheet on the computational domain plane z=0.445 is shown in Fig. 2f. The thin red line in Fig. 2d-f is the line of the constant current density. Fig. 2g-j presents the projections of magnetic lines on the plane that superimposed on the X-ray intensity maps in different ranges with magnetograms.

Fig. 3 present the magnetic lines passing through the points located near the current sheet center in the plane of configuration w=0 and in the planes parallel to it w=-0.005 and w=0.002. The arrangement of these lines in space and their projections on the plane of the configuration and on picture plane are shown. The locations of all lines besides one of them in the corona above the active region are similar to the location of line passing through the current sheet center (Fig. 2e). The location in the corona of one line passing through the point in the plane w=-0.005 (the point is (u, v, w)=(0.007, 0.013, -0.005)) essentially differs from the location of other lines. This line is shown as a bold red one in the front of the plane w=-0.005, and this line is shown as a thin brown one behind this plane in Fig. 3a, b, c. This line is shown in the computational domain in corona in Fig. 4a. Its projections on the plane z=0.445 (Fig. 4b) and on the picture plane together with the map of hard X-ray emission imposed on magnetogram (Fig. 4c) are shown. This line is shown as a bold red one in the front of the plane z=0.445, and this line is shown as a thin brown one behind this plane in Fig. 4a, b, c. This magnetic line intersects the photosphere at a distance of 18'' (~12 600 km or ~0.03 of dimensionless unit) from the source of hard X-ray emission (Fig. 4c). The magnetic field near the photosphere can be distorted due to numerical instabilities; therefore it is difficult to say now how accurately crossing points of the magnetic lines, issued from the current sheet, correspond to crossing points of real magnetic field. Apparently, the accuracy of the calculation can not precisely reproduce the shape of the field lines which crosses the current sheet, shown on Figures 2 and 3. Thus lines do not cross the photosphere in the locations of sources of hard X-ray emission. However, the appearance of a line, passing close to the current sheet for the carried out calculation indicates the possibility of the existence of a real magnetic field line crossing the current sheet, which intersect the photosphere at the location of the hard X-ray sources.

Conclusions

1. The system of graphical output for the study of the magnetic field complex configuration in the corona above the active region during the flare situation was developed and modernized.
2. The study of the magnetic field configuration near the current sheet in the corona above the active region 10365 flare May 27, 2003 at 02:53 showed that the physical meaning of the processes of accumulation and rapid release of the flare energy is best presented by the lines in the plane of the current sheet configuration which are tangential to the projections of the magnetic field vectors on this plane. From the picture of these lines it is easy to understand directions of magnetic $\mathbf{j}\times\mathbf{B}/c$ forces which are perpendicular to these lines. The j×B/c forces cause the plasma motion toward and away from the singular line, movements that deforms the magnetic field into the current sheet configuration
3. The magnetic field configuration near the current sheet is complicated and it is difficult to understand the magnetic forces direction by analyzing the behavior of magnetic lines. However, the behavior of magnetic lines can be studied in detail using the developed graphical system which permits to present the line in arbitrary orientated subregion and to present the line projection on arbitrary plane.
4. Locations of the magnetic lines in the corona passing close to the current sheet and crossing the photosphere are analyzed. Since the calculated magnetic field is distorted near the photosphere due to numerical instabilities, the study only allows us to draw a preliminary conclusion about the possibility of crossing the photosphere by the magnetic line, arriving out of the current sheet, to the source of hard X-ray location. More accurate conclusions can be made after the calculation in real time scale, in which the instability near the photospheric boundary associated with abnormally rapid change of the magnetic field should be significantly suppressed.

Acknowledgments

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