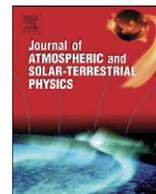




Contents lists available at ScienceDirect

# Journal of Atmospheric and Solar-Terrestrial Physics

journal homepage: [www.elsevier.com/locate/jastp](http://www.elsevier.com/locate/jastp)

## A new version of the NeQuick ionosphere electron density model

B. Nava\*, P. Coisson, S.M. Radicella

Aeronomy and Radiopropagation Laboratory, The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy

### ARTICLE INFO

#### Article history:

Accepted 29 January 2008

Available online 5 February 2008

#### Keywords:

Ionospheric modeling

Topside region

F region

Total electron content

### ABSTRACT

NeQuick is a three-dimensional and time dependent ionospheric electron density model developed at the Aeronomy and Radiopropagation Laboratory of the Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy and at the Institute for Geophysics, Astrophysics and Meteorology of the University of Graz, Austria. It is a quick-run model particularly tailored for trans-ionospheric applications that allows one to calculate the electron concentration at any given location in the ionosphere and thus the total electron content (TEC) along any ground-to-satellite ray-path by means of numerical integration. Taking advantage of the increasing amount of available data, the model formulation is continuously updated to improve NeQuick capabilities to provide representations of the ionosphere at global scales. Recently, major changes have been introduced in the model topside formulation and important modifications have also been introduced in the bottomside description. In addition, specific revisions have been applied to the computer package associated to NeQuick in order to improve its computational efficiency. It has therefore been considered appropriate to finalize all the model developments in a new version of the NeQuick. In the present work the main features of NeQuick 2 are illustrated and some results related to validation tests are reported.

© 2008 Elsevier Ltd. All rights reserved.

### 1. Introduction

The NeQuick (Hochegger et al., 2000; Radicella and Leitinger, 2001) is an ionospheric electron density model developed at the Aeronomy and Radiopropagation Laboratory of The Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy, and at the Institute for Geophysics, Astrophysics and Meteorology (IGAM) of the University of Graz, Austria. Historically the NeQuick has to be considered as an evolution of the DGR profiler proposed by Di Giovanni and Radicella (1990), and subsequently modified by Radicella and Zhang (1995).

The first version of the model has been used by the European Space Agency (ESA) European Geostationary Navigation Overlay Service (EGNOS) project for assessment

analysis and has been adopted for single-frequency positioning applications in the framework of the European Galileo project. It has also been adopted by the International Telecommunication Union, Radiocommunication Sector (ITU-R) as a suitable method for total electron content (TEC) modeling (ITU, 2003). In addition, the NeQuick has been implemented in the simulation toolkit developed in Australia to conduct a qualitative assessment of the performance characteristics of the future GNSS infrastructure (Seynat et al., 2004) and it has been adapted by the Rutherford-Appleton Laboratory of the UK to forecast vertical TEC from forecasted values of  $f_oF_2$  and  $MUF(3000)F_2$  (Cander, 2003). NeQuick (FORTRAN 77) source code is available at <http://www.itu.int/ITU-R/index.asp?category=documents&link=rsg3&lang=en>.

As in the case of other models, like for example the IRI (Bilitza, 2001), many efforts have been done to improve the NeQuick analytical formulation and taking advantage of the increasing amount of available data, the NeQuick has been continuously updated. In particular

\* Corresponding author. Tel.: +39 40 2240340; fax: +39 40 224604.

E-mail addresses: [bnava@ictp.it](mailto:bnava@ictp.it) (B. Nava), [coissonp@ictp.it](mailto:coissonp@ictp.it) (P. Coisson), [rsandro@ictp.it](mailto:rsandro@ictp.it) (S.M. Radicella).

the modifications have been done taking into account the necessity for the model to provide better representations of the median ionosphere at global scales. Recently major changes have been introduced in the bottomside (Leitinger et al., 2005) and in the topside (Coïsson et al., 2006) description of the model. In addition, specific revisions have been applied to the computer package associated with NeQuick model in order to improve its computational efficiency. All these efforts, directed toward the developments of a new version of the model, have therefore led to the implementation of the NeQuick 2. In the following paragraphs the main features of the new version of the NeQuick will be described and the analytical formulation of the model will be reported.

## 2. The NeQuick 2

Being the version 2 of the NeQuick an evolution of the version 1, the conceptual structure of the model has been kept unchanged. Nevertheless, the formulation of some specific parameter related to the bottomside and topside description has been modified.

In the following, only the main features of the NeQuick 2 will be indicated, and the complete analytical formulation of the model will be given subsequently.

To describe the electron density of the ionosphere above 90 km and up to the peak of the  $F_2$  layer the NeQuick 2 uses a modified DGR profile formulation (Di Giovanni and Radicella, 1990), which includes five semi-Epstein layers (Rawer, 1982) with modeled thickness parameters (Radicella and Zhang, 1995). Three profile anchor points are used; namely the  $E$  layer peak, the  $F_1$  peak and the  $F_2$  peak that are modeled in terms of the ionosonde parameters  $foE$ ,  $foF_1$ ,  $foF_2$  and  $M(3000)F_2$ . These values can be modeled, as indicated in Leitinger et al. (2005), or experimentally derived. The model topside is represented by a semi-Epstein layer with a height-dependent thickness parameter (Hochegger et al., 2000) empirically determined (Coïsson et al., 2006).

The basic inputs of the NeQuick model are: position, time and solar flux (or sunspot number); the output is the electron concentration at the given location and time.

As in the case of the previous version, the NeQuick 2 computer package includes specific routines to evaluate the electron density along any ground-to-satellite ray-path and the corresponding TEC by numerical integration.

### 2.1. The NeQuick 2 analytical formulation

Before describing the NeQuick 2 in detail, we recall that an Epstein layer (Rawer, 1982) can be represented by

$$N_{\text{Epstein}}(h; hmax, Nmax, B) = \frac{4Nmax}{\left(1 + \exp\left(\frac{h - hmax}{B}\right)\right)^2} \exp\left(\frac{h - hmax}{B}\right), \quad (1)$$

where  $Nmax$  is the layer peak electron density,  $hmax$  is the layer peak height and  $B$  is the layer thickness parameter.

### 2.1.1. The bottomside formulation

Using the expressions  $NmE = 0.124(foE)^2$ ,  $NmF_1 = 0.124(foF_1)^2$ ,  $NmF_2 = 0.124(foF_2)^2$  for the  $E$ ,  $F_1$  and  $F_2$  layer peak electron densities (in  $10^{11} \text{ m}^{-3}$ ), respectively,  $hmE$ ,  $hmF_1$ ,  $hmF_2$  for the  $E$ ,  $F_1$  and  $F_2$  layer peak heights (in km), respectively, and  $BE$ ,  $B_1$ ,  $B_2$  for the  $E$ ,  $F_1$  and  $F_2$  layer thickness parameters (in km), respectively, the bottomside of the NeQuick 2 can be expressed as a sum of semi-Epstein layers as follows:

$$N_{\text{bot}}(h) = N_E(h) + N_{F_1}(h) + N_{F_2}(h), \quad (2)$$

where

$$N_E(h) = \frac{4Nm^*E}{\left(1 + \exp\left(\frac{h - hmE}{BE} \xi(h)\right)\right)^2} \times \exp\left(\frac{h - hmE}{BE} \xi(h)\right), \quad (3)$$

$$N_{F_1}(h) = \frac{4Nm^*F_1}{\left(1 + \exp\left(\frac{h - hmF_1}{B_1} \xi(h)\right)\right)^2} \times \exp\left(\frac{h - hmF_1}{B_1} \xi(h)\right), \quad (4)$$

$$N_{F_2}(h) = \frac{4NmF_2}{\left(1 + \exp\left(\frac{h - hmF_2}{B_2}\right)\right)^2} \times \exp\left(\frac{h - hmF_2}{B_2}\right) \quad (5)$$

with

$$Nm^*E = NmE - N_{F_1}(hmE) - N_{F_2}(hmE), \quad (6)$$

$$Nm^*F_1 = NmF_1 - N_E(hmF_1) - N_{F_2}(hmF_1) \quad (7)$$

and

$$\xi(h) = \exp\left(\frac{10}{1 + 1|h - hmF_2|}\right) \quad (8)$$

is a function that ensures a “fadeout” of the  $E$  and  $F_1$  layers in the vicinity of the  $F_2$  layer peak in order to avoid secondary maxima around  $hmF_2$ . In accordance to the behavior of the  $F_1$  layer, expression (6) and (7) can be slightly modified.

The thickness parameters take different values for the bottomside and for the topside of each layer ( $BE_{\text{bot}}$  and  $BE_{\text{top}}$  for the  $E$  layer,  $B_{1\text{bot}}$  and  $B_{1\text{top}}$  for the  $F_1$  layer,  $B_{2\text{bot}}$  for the  $F_2$  layer).

### 2.1.2. The topside formulation

The model topside is represented by a semi-Epstein layer with a height-dependent thickness parameter  $H$ :

$$N(h) = \frac{4NmF_2}{(1 + \exp(z))^2} \exp(z) \quad (9)$$

with

$$z = \frac{h - hmF_2}{H}, \quad (10)$$

$$H = H_0 \left[ 1 + \frac{rg(h - hmF2)}{rH_0 + g(h - hmF2)} \right], \quad (11)$$

where the constant parameters

$$r = 100, \quad (12)$$

$$g = 0.125 \quad (13)$$

are used to control the increase of  $H$ . The parameter  $H_0$  will be specified in Section 2.2.2.

## 2.2. Parameter modeling

In the present section all the expressions used to evaluate the parameters needed to compute a vertical electron density profile will be given.

### 2.2.1. Peak heights

The heights in km of the  $E$ ,  $F1$  and  $F2$  layer maximum densities are given by

$$hmE = 120, \quad (14)$$

$$hmF1 = \frac{hmE + hmF2}{2}, \quad (15)$$

$$hmF2 = \frac{1490MF}{M + \Delta M} - 176, \quad (16)$$

where

$$\Delta M = \begin{cases} 0.253/(foF2/foE \\ -1.215) - 0.012, \\ -0.012 \end{cases} \quad \text{if } foE = 0, \quad (17)$$

$$MF = M \sqrt{\frac{0.0196M^2 + 1}{1.2967M^2 - 1}} \quad (18)$$

and

$$M = M(3000)F2. \quad (19)$$

The formula for  $hmF1$  is one of the changes specifically introduced in the NeQuick 2 bottomside (Leitinger et al., 2005), whereas the equations for  $hmF2$  (Radicella and Zhang, 1995) are based on the Dudeney (1978, 1983) formula for the peak electron density height, as in the previous version of the model.

### 2.2.2. Thickness parameters

The semi-thickness parameter  $BE_{\text{bot}}$  and  $BE_{\text{top}}$  (for the  $E$  layer),  $B1_{\text{bot}}$  and  $B1_{\text{top}}$  (for the  $F1$  layer) and  $B2_{\text{bot}}$  and  $H$  (for the  $F2$  layer) are given in km and expressed by the following relations:

$$BE_{\text{bot}} = 5, \quad (20)$$

$$BE_{\text{top}} = \max(0.5(hmF1 - hmE), 7), \quad (21)$$

$$B1_{\text{bot}} = 0.5(hmF1 - hmE), \quad (22)$$

$$B1_{\text{top}} = 0.3(hmF2 - hmF1), \quad (23)$$

$$B2_{\text{bot}} = \frac{0.385NmF2}{(dN/dh)_{\text{max}}}, \quad (24)$$

$$H = kB2_{\text{bot}} \left[ 1 + \frac{rg(h - hmF2)}{rkB2_{\text{bot}} + g(h - hmF2)} \right]. \quad (25)$$

In particular, relations (21)–(23) are the result of an elaborate revision (Leitinger et al., 2005) of the  $BE_{\text{top}}$ ,  $B1_{\text{bot}}$ ,  $B1_{\text{top}}$  formulation adopted in the previous version of the model (Radicella and Zhang, 1995).

Expression (24) depends on the value of the maximum of the electron density derivative with respect to height. This maximum is computed from  $foF2$  and  $M(3000)F2$  values, using the empirical relation (Mosert de Gonzales and Radicella, 1990) given as

$$\ln \left( \left( \frac{dN}{dh} \right)_{\text{max}} \right) = -3.467 + 1.714 \ln(foF2) + 2.02 \ln(M(3000)F2), \quad (26)$$

where  $dN/dh$  is in ( $10^9 \text{ m}^{-3} \text{ km}^{-1}$ ) and  $foF2$  in (MHz).

Expression (25) is the same as (11) with  $H_0 = kB2_{\text{bot}}$ .

The parameter  $k$ , which appears in Eq. (25), is given by Coisson et al. (2006)

$$k = 3.22 - 0.0538foF2 - 0.00664hmF2 + 0.113 \frac{hmF2}{B2_{\text{bot}}} + 0.00257R12, \quad (27)$$

where  $hmF2$  (km),  $foF2$  (MHz) are the  $F2$  layer peak parameters,  $B2_{\text{bot}}$  (km) the thickness of the  $F2$  bottomside and  $R12$  the smoothed sunspot number. As inferred from the experimental data analysis, the restriction  $k \geq 1$  is applied in the model.

It has to be noted that the new formulation (27) of the parameter  $k$  constitutes one of the major changes introduced in the new version of the model.

### 2.2.3. Critical frequencies and propagation factor

Taking into account that the NeQuick model has been designed mostly for trans-ionospheric propagation applications, the representation of the lower part of the ionosphere has been kept as simple as possible.

The Titheridge model for  $foE$  (Leitinger et al., 1995; Titheridge, 1996) has been adopted. It is based on the seasonal relationship between the solar zenith angle  $\chi$  and  $foE$  given as

$$(foE)^2 = (a_e \sqrt{F107})^2 (\cos \chi_{\text{eff}})^{0.6}, \quad (28)$$

where  $a_e$  is the seasonal term represented in the Table 1,  $F107$  is the 10.7 cm solar radio noise flux and  $\chi_{\text{eff}}$  is the solar zenith angle:

$$\chi_{\text{eff}} = \chi \quad \text{when } \chi \leq 86.23^\circ, \quad (29)$$

$$\chi_{\text{eff}} = 90^\circ - 0.24^\circ \exp(20^\circ - 0.2\chi) \quad \text{when } \chi > 86.23^\circ. \quad (30)$$

**Table 1**

Seasonal term to compute  $foE$  in the Titheridge's model (28) for the Northern and Southern hemisphere

$a_e$	Month North	Month South
1.131	1, 2, 11, 12	5, 6, 7, 8
1.112	3, 4, 9, 10	3, 4, 9, 10
1.093	5, 6, 7, 8	1, 2, 11, 12

Eq. (29) is used during daytime and (30) during nighttime. An exponential day–night transition is used to ensure the continuity of  $foE$  and its first derivative at the solar terminator.

Following Leitinger et al. (2005),  $foF1$  is related to  $foE$  by

$$foF1 = \begin{cases} 1.4foE & \text{if } foE \geq 2, \\ 0 & \text{if } foE < 2, \\ 0.85 * 1.4foE & \text{if } 1.4foE > 0.85foF2 \end{cases} \quad (31)$$

while in the previous version of the model, the  $F1$  layer was vanishing during the night and it was simply  $foF1 = 1.4foF2$  during the day.

To compute median values of  $foF2$  and  $M(3000)F2$ , the NeQuick model uses the ITU-R (formerly called CCIR, 1967) coefficients (Jones and Gallet, 1962, 1965). It is important to note that the computation of  $foF2$  and  $M(3000)F2$  at a given location requires the knowledge of the modip (Rawer, 1963) of the same location. The NeQuick package therefore includes a set of 12 files, each one containing the coefficients for both  $foF2$  and  $M(3000)F2$  for one month of the year, and a file containing the modip values for a world wide grid with a spacing of  $5^\circ$  in latitude and  $10^\circ$  in longitude.

The specific functions and interpolation routines to obtain the required parameter values are also included in the computer program associated to the NeQuick 2 model.

2.3. Source code

Considering the necessity of being a quick-run model, specific revisions have been applied also to the computer package associated to NeQuick 2 model in order to improve its computational efficiency. In this new model, the change in the formulation of the  $F1$  layer, now allows one to compute only the values of modip (avoiding the computation of dip latitude as in the previous version).

In addition, we recall that the modip grid included in the NeQuick 2 package corresponds to the geomagnetic

field given by the IGRF model for the year 2005. Nevertheless, it is possible to change the geomagnetic field configuration by changing the modip file accordingly.

The new source code (written in FORTRAN 77) also includes some technical changes (the “ENTRY” statements have been avoided) and it can be easily adapted to accept grids of driving parameter as inputs, following the concepts expressed by Leitinger et al. (2001).

Some formal bugs in the former version of the model have been eliminated in this new one.

3. Model validation

In the present paragraph the main reasons that lead to the major changes in the NeQuick 2 are indicated and the corresponding improvements obtained in terms of ionosphere electron density representation are summarized.

As indicated by Leitinger et al. (2005), when the first version of NeQuick is used to compute electron density grids at fixed heights below the  $F2$  peak, in some cases strong gradients and strange structures appear in  $E$  and  $F1$  layer heights. The strategy used to solve these problems is extensively described in the same paper. Here, an indication of the results achieved after the model revision is given with an example. Fig. 1 represents an electron density grid computed at 200 km of height using the version 1 of the NeQuick driven by the following inputs: solar flux = 122 flux units (f.u.), with  $1 \text{ f.u.} = 10^{-22} \text{ W m}^{-2} \text{ Hz}^{-1}$ ; month = 11; UT = 11. The electron density isolines, also illustrated in the figure, clearly outline an unrealistic electron density structure in the diagonal of the display.

For comparison, Fig. 2 represents the same electron density grid computed at 200 km of height, but using the version 2 of the NeQuick driven by the same input parameters utilized to construct the grid in Fig. 1. It is evident that in the grid calculated with NeQuick 2 the previously mentioned electron density structure has disappeared.

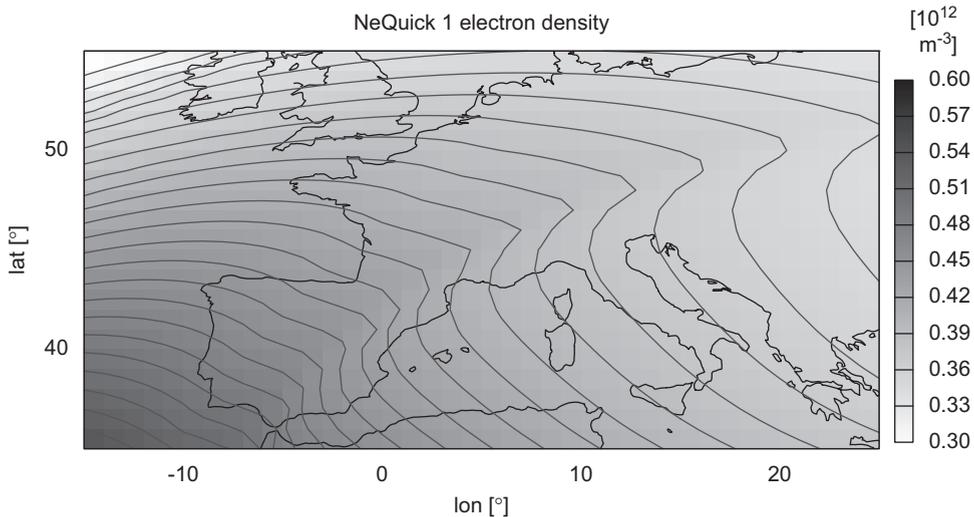


Fig. 1. NeQuick: electron density map at 200 km of height for November, 1100 UT, 122 f.u. Isolines of electron density are also indicated.

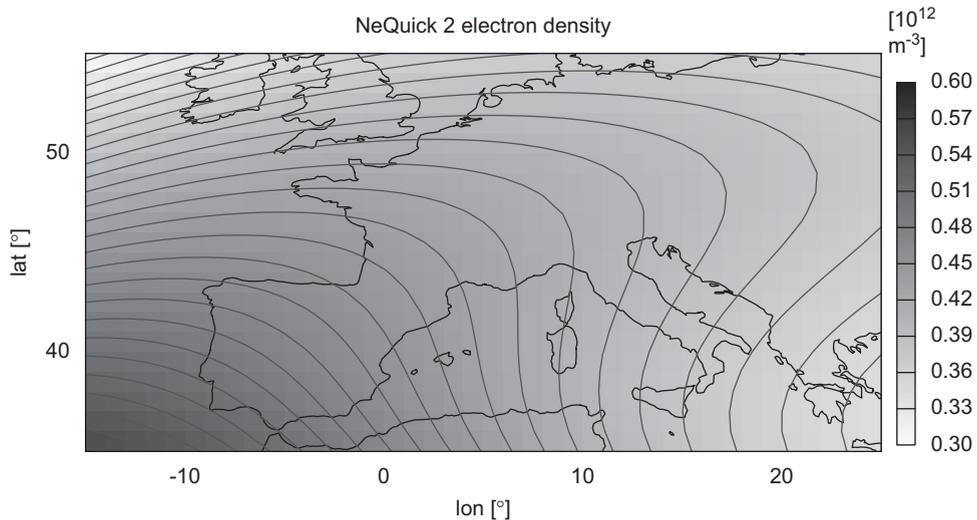


Fig. 2. NeQuick 2: electron density map at 200 km of height for November, 1100 UT, 122 f.u. Isolines of electron density are also indicated.

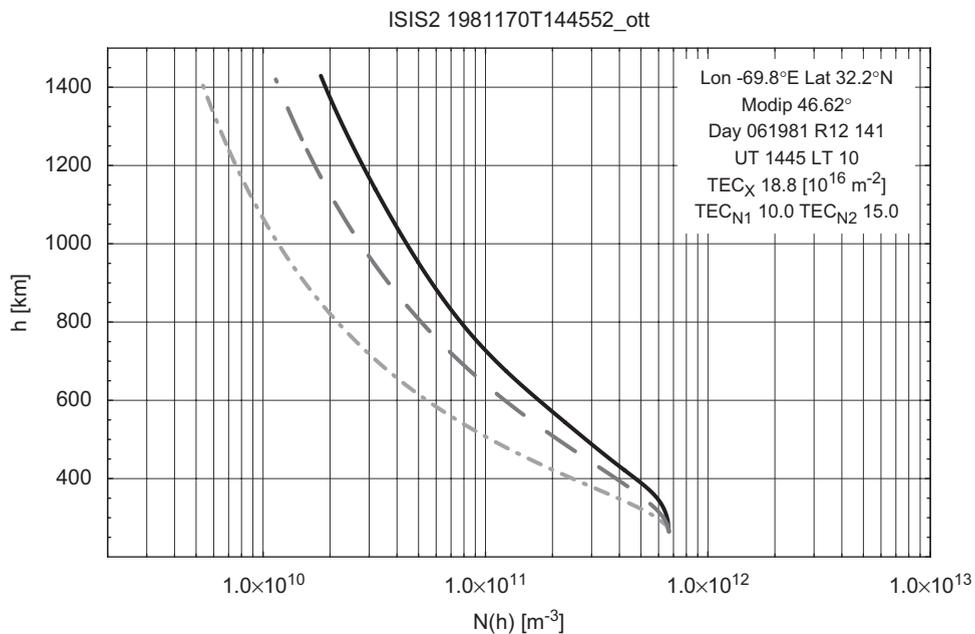


Fig. 3. Example of topside electron density profiles comparison. Solid line: experimental; dashed-dotted line: NeQuick; dashed line: NeQuick 2.

A major revision of the topside formulation implied the redefinition of the shape parameter  $k$  (see expression (27)) on the bases of new available experimental topside sounder data (Coisson et al., 2006). The revised expression introduces also a simplification because in the previous version the parameter  $k$  had a twofold seasonal dependence (Radicella and Zhang, 1995). As an example, in Fig. 3 the experimental as well as the modeled topside profiles (using NeQuick and NeQuick 2) are represented. In particular the modeled profiles are anchored to the experimental peak parameter values. These values have

been obtained by the ISIS2 satellite on 19 June 1981 for the location having latitude of  $32.2^\circ\text{N}$  and a longitude of  $69.8^\circ\text{W}$  during a period of high solar activity. It has to be mentioned that the experimental topside profile used for the comparison has not been used in the derivation of expression (27). In order to illustrate the global differences between the previous and the new version of the NeQuick model, the vertical TEC maps in Figs. 4 and 5 are considered as an example. The map in Fig. 4 has been computed using the NeQuick model; the map in Fig. 5 has been calculated utilizing the NeQuick 2. The same inputs

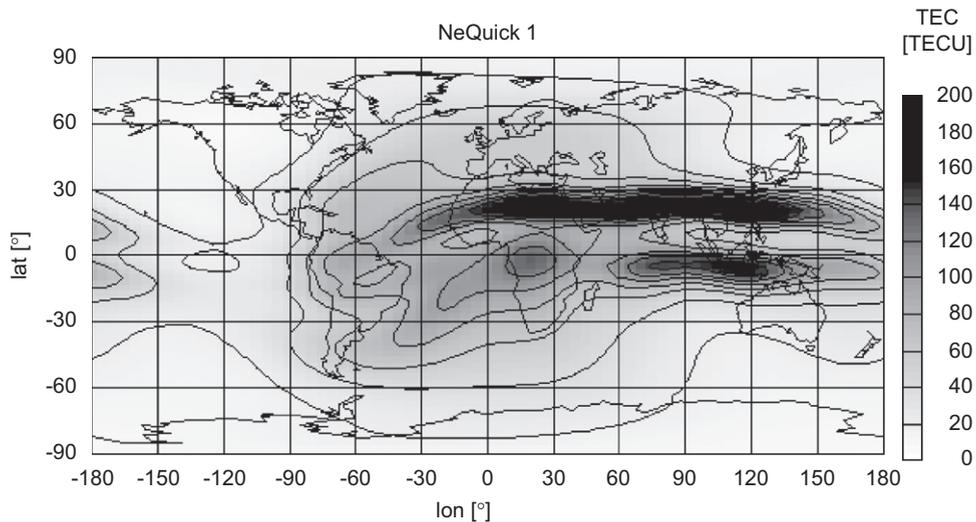


Fig. 4. NeQuick 1: vertical TEC map for the month of October, 1300 UT, 190 f.u.

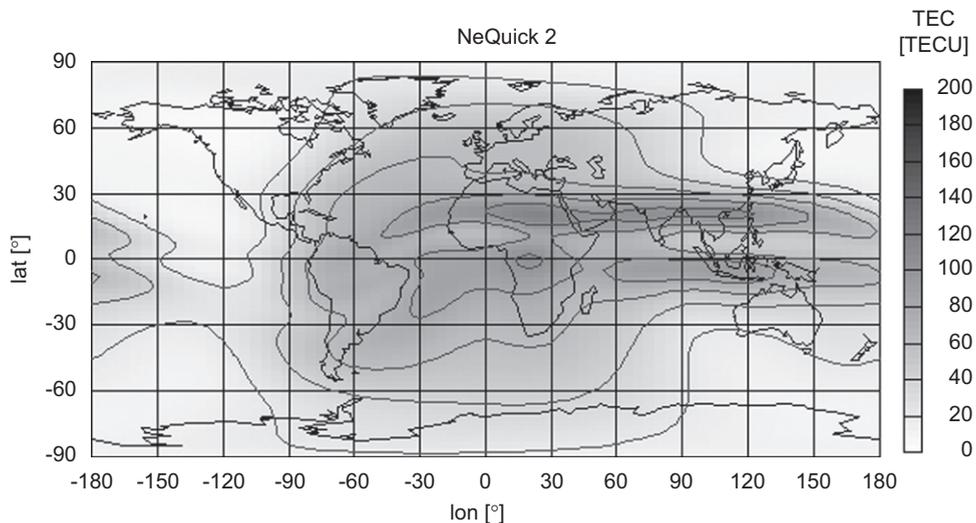


Fig. 5. NeQuick 2: vertical TEC map for the month of October, 1300 UT, 190 f.u.

(solar flux = 190 f.u.; month = 10; UT = 13) have been used for both model versions. As can be seen from Fig. 4, in some cases the TEC can exceed the 200 TECU ( $1\text{TECU} = 10^{16} \text{m}^{-2}$ ), and reach median values that can be considered unrealistic for the given conditions. In Fig. 5 the TEC never exceeds the more realistic value of 150 TECU. This confirms that the NeQuick 2 is able to better represent the behavior of a median ionosphere at global scales.

It must be mentioned that some preliminary works have been done in order to implement electron density retrieval techniques (like those illustrated in Nava et al., 2006) based on NeQuick 2 adaptation to experimental data. The first results confirm that the new version of the model performs better when it is used to reproduce actual conditions of the ionosphere.

#### 4. Conclusions

In recent years several changes have been introduced in the version 1 of the NeQuick model. The most important modifications are related to the bottomside formulation in terms of the modeling of the F1 layer peak electron density, height and thickness parameter.

Concerning the model topside, a new formulation of the shape parameter  $k$  has been adopted.

All the model improvements have therefore been considered to finalize a new version of the model: the NeQuick 2.

Correspondingly, the computer package associated with the analytical model has been modified and specific optimizations have been implemented to improve its computational efficiency.

Being coherent with the philosophy that has been followed until now, the NeQuick 2 will be continuously tested and evaluated. It is therefore expected that possible further improvement will be introduced in the future.

## Acknowledgments

The authors are grateful to Prof. Reinhart Leitinger of the University of Graz, Austria, who has strongly contributed to the development of the NeQuick model and to all its implementations.

## References

- Bilitza, D., 2001. International reference ionosphere 2000. *Radio Science* 36 (2), 261–275.
- Cander, L.J.R., 2003. Towards forecasting and mapping ionospheric space weather under COST actions. *Advances in Space Research* 31 (4), 957–964.
- CCIR, 1967. Atlas of ionospheric characteristics. Comité Consultatif International des Radiocommunications, Report 340-4, ITU, Geneva.
- Coisson, P., Radicella, S.M., Leitinger, R., Nava, B., 2006. Topside electron density in IRI and NeQuick: features and limitations. *Advances in Space Research* 37, 937–942.
- Di Giovanni, G., Radicella, S.M., 1990. An analytical model of the electron density profile in the ionosphere. *Advances in Space Research* 10 (11), 27–30.
- Dudeney, J.R., 1978. An improved model of the variation of electron concentration with height in the ionosphere. *Journal of Atmospheric and Terrestrial Physics* 40, 195–203.
- Dudeney, J.R., 1983. The accuracy of simple methods for determining the height of the maximum electron concentration of the F2-layer from scaled ionospheric characteristics. *Journal of Atmospheric and Terrestrial Physics* 45, 629–640.
- Jones, W.B., Gallet, R.M., 1962. Representation of diurnal and geographic variations of ionospheric data by numerical methods. *ITU Telecommunication Journal* 29 (5), 129–149.
- Jones, W.B., Gallet, R.M., 1965. Representation of diurnal and geographic variations of ionospheric data by numerical methods. *ITU Telecommunication Journal* 32, 18–29.
- Hochegger, G., Nava, B., Radicella, S.M., Leitinger, R., 2000. A family of ionospheric models for different uses. *Physics and Chemistry of the Earth, Part C: Solar, Terrestrial & Planetary Science* 25 (4), 307–310.
- ITU, 2003. Ionospheric propagation data and prediction methods required for the design of satellite services and systems. Recommendation P. 531-7, Geneva.
- Leitinger, R., Titheridge, J.E., Kirchengast, G., Rothleitner, W., 1995. A "simple" global empirical model for the F layer of the ionosphere. *Wissenschaftliche Berichte* 1/1995, IMG, University of Graz.
- Leitinger, R., Nava, B., Hochegger, G., Radicella, S.M., 2001. Ionospheric profiles using data grids. *Physics and Chemistry of the Earth (C)* 26 (5), 293–301.
- Leitinger, R., Zhang, M.L., Radicella, S.M., 2005. An improved bottomside for the ionospheric electron density model NeQuick. *Annals of Geophysics* 48 (3), 525–534.
- Mosert de Gonzales, M., Radicella, S.M., 1990. On a characteristic point at the base of F2 layer in the ionosphere. *Advances in Space Research* 10 (11), 17–25.
- Nava, B., Radicella, S.M., Leitinger, R., Coisson, P., 2006. A near-real-time model-assisted ionosphere electron density retrieval method. *Radio Science* 41, RS6S16.
- Radicella, S.M., Leitinger, R., 2001. The evolution of the DGR approach to model electron density profiles. *Advances in Space Research* 27 (1), 35–40.
- Radicella, S.M., Zhang, M.L., 1995. The improved DGR analytical model of electron density height profile and total electron content in the ionosphere. *Annali di Geofisica XXXVIII* (1), 35–41.
- Rawer, K., 1963. In: Landmark, B. (Ed.), *Meteorological and Astronomical Influences on Radio Wave Propagation*. Academic Press, New York, pp. 221–250.
- Rawer, K., 1982. Replacement of the present sub-peak plasma density profile by a unique expression. *Advances in Space Research* 2 (10), 183–190.
- Seynat, C., Kealy, A., Zhang, K., 2004. A performance analysis of future global navigation satellite systems. *Journal of Global Positioning Systems* 3 (1–2), 232–241.
- Titheridge, J.E., 1996. Re-modeling the ionospheric E region. *Kleinhebacher Berichte* 39, 687–696.