Physics of motion of charges in EM fields for understanding MAGDAS and FMCW radar data

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Outline

- Geometric Algebra
- Maxwell's Equation
- Wave Equation
- Electromagnetic Waves
- Wave Equation in Plasma
- Refractive Index of plasma
- FMCW radar wave propagation
- Ionogram simulations

Philippine MAGDAS Network

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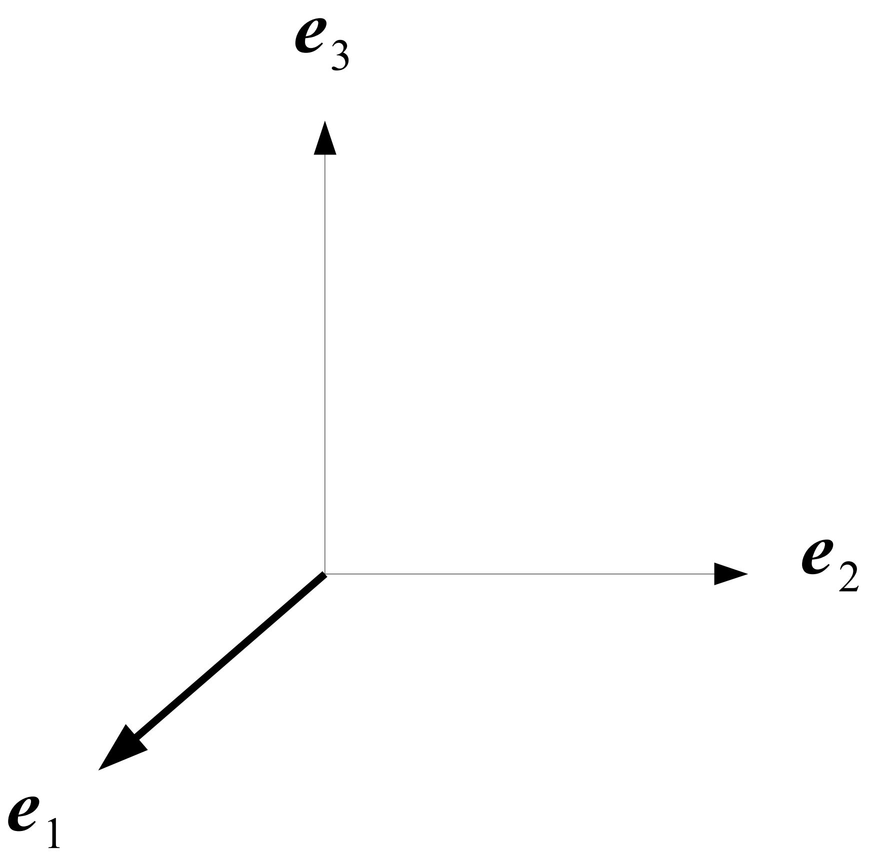
Ionosphere Research Building: ICSWSE Subcenter

Vector algebra



$$e_j e_k + e_k e_j = 2\delta_{jk}$$

$$e_1^2 = e_2^2 = e_3^2 = 1$$

$$e_1 e_2 = -e_2 e_1$$
$$e_2 e_3 = -e_3 e_2$$
$$e_3 e_1 = -e_1 e_3$$

Unit vectors square to +1
Perpendicular vectors anticommute

Imaginary numbers: bivectors

$$(\boldsymbol{e}_1\,\boldsymbol{e}_2)^2 = (\boldsymbol{e}_1\,\boldsymbol{e}_2)(\boldsymbol{e}_1\,\boldsymbol{e}_2) = \boldsymbol{e}_1\,\boldsymbol{e}_2\,\boldsymbol{e}_1\,\boldsymbol{e}_2$$

$$= \boldsymbol{e}_1\,(\boldsymbol{e}_2\,\boldsymbol{e}_1)\,\boldsymbol{e}_2 = \boldsymbol{e}_1\,(-\boldsymbol{e}_1\,\boldsymbol{e}_2)\,\boldsymbol{e}_2$$

$$= -\boldsymbol{e}_1\,\boldsymbol{e}_1\,\boldsymbol{e}_2\,\boldsymbol{e}_2 = -(\boldsymbol{e}_1\,\boldsymbol{e}_1)\,(\boldsymbol{e}_2\,\boldsymbol{e}_2)$$

$$= -(1)(1) = -1$$

$$(\boldsymbol{e}_1\,\boldsymbol{e}_2)^2 = (\boldsymbol{e}_2\,\boldsymbol{e}_3)^2 = (\boldsymbol{e}_3\,\boldsymbol{e}_1)^2 = -1$$

Imaginary numbers: bivectors

$$(\mathbf{e}_1\,\mathbf{e}_2)^2 = (\mathbf{e}_1\,\mathbf{e}_2)(\mathbf{e}_1\,\mathbf{e}_2) = \mathbf{e}_1\,\mathbf{e}_2\,\mathbf{e}_1\,\mathbf{e}_2$$

$$= \mathbf{e}_1\,(\mathbf{e}_2\,\mathbf{e}_1)\,\mathbf{e}_2 = \mathbf{e}_1\,(-\mathbf{e}_1\,\mathbf{e}_2)\,\mathbf{e}_2$$

$$= -\mathbf{e}_1\,\mathbf{e}_1\,\mathbf{e}_2\,\mathbf{e}_2 = -(\mathbf{e}_1\,\mathbf{e}_1)\,(\mathbf{e}_2\,\mathbf{e}_2)$$

$$= -(1)(1) = -1$$

$$(\mathbf{e}_1\,\mathbf{e}_2)^2 = (\mathbf{e}_2\,\mathbf{e}_3)^2 = (\mathbf{e}_3\,\mathbf{e}_1)^2 = -1$$

Imaginary number: trivector

$$i = \boldsymbol{e}_1 \boldsymbol{e}_2 \boldsymbol{e}_3$$

1

$$i^2 = \boldsymbol{e}_1 \boldsymbol{e}_2 \boldsymbol{e}_3 \boldsymbol{e}_1 \boldsymbol{e}_2 \boldsymbol{e}_3 = - - \boldsymbol{e}_1 \boldsymbol{e}_1 \boldsymbol{e}_2 \boldsymbol{e}_3 \boldsymbol{e}_2 \boldsymbol{e}_3$$

$$= \boldsymbol{e}_2 \boldsymbol{e}_3 \boldsymbol{e}_2 \boldsymbol{e}_3 = -1$$

$$i^2 = -1$$

Imaginary vector is a bivector

$$i\boldsymbol{e}_1 = \boldsymbol{e}_1\boldsymbol{e}_2\boldsymbol{e}_3\boldsymbol{e}_1 = \boldsymbol{e}_2\boldsymbol{e}_3 = -\boldsymbol{e}_3\boldsymbol{e}_2 = \boldsymbol{e}_1 i$$

$$i\boldsymbol{e}_2 = \boldsymbol{e}_1\boldsymbol{e}_2\boldsymbol{e}_3\boldsymbol{e}_2 = -\boldsymbol{e}_1\boldsymbol{e}_3 = \boldsymbol{e}_3\boldsymbol{e}_1 = \boldsymbol{e}_2 i$$

$$i\boldsymbol{e}_3 = \boldsymbol{e}_1\boldsymbol{e}_2\boldsymbol{e}_3\boldsymbol{e}_3 = \boldsymbol{e}_1\boldsymbol{e}_2 = -\boldsymbol{e}_2\boldsymbol{e}_1 = \boldsymbol{e}_3 i$$

Note: imaginary number i commutes with vectors

Geometric product

$$\boldsymbol{a}\boldsymbol{b} = (a_1\boldsymbol{e}_1 + a_2\boldsymbol{e}_2 + a_3\boldsymbol{e}_3)(b_1\boldsymbol{e}_1 + b_2\boldsymbol{e}_2 + b_3\boldsymbol{e}_3)$$

$$\boldsymbol{a}\boldsymbol{b} = a_1 b_1 \boldsymbol{e}_1\boldsymbol{e}_1 + a_1 b_2 \boldsymbol{e}_1\boldsymbol{e}_2 + a_1 b_3 \boldsymbol{e}_1\boldsymbol{e}_3 +$$

$$a_2 b_1 \boldsymbol{e}_2\boldsymbol{e}_1 + a_2 b_2 \boldsymbol{e}_2\boldsymbol{e}_2 + a_2 b_3 \boldsymbol{e}_2\boldsymbol{e}_3 +$$

$$a_3 b_1 \boldsymbol{e}_3\boldsymbol{e}_2 + a_3 b_2 \boldsymbol{e}_3\boldsymbol{e}_2 + a_3 b_3 \boldsymbol{e}_3\boldsymbol{e}_3$$

$$\boldsymbol{a}\boldsymbol{b} = \boldsymbol{a}\cdot\boldsymbol{b} + i(\boldsymbol{a}\times\boldsymbol{b})$$

Cross product

Dot product

Dot and Cross Products

$$\boldsymbol{a}\,\boldsymbol{b} = \boldsymbol{a}\cdot\boldsymbol{b} + i\left(\boldsymbol{a}\times\boldsymbol{b}\right)$$

$$\boldsymbol{b}\,\boldsymbol{a} = \boldsymbol{b}\cdot\boldsymbol{a} + i\left(\boldsymbol{b}\times\boldsymbol{a}\right)$$

$$\boldsymbol{a}\cdot\boldsymbol{b} = \boldsymbol{b}\cdot\boldsymbol{a}$$

$$\boldsymbol{a}\times\boldsymbol{b} = -\boldsymbol{b}\times\boldsymbol{a}$$

$$\boldsymbol{a}\cdot\boldsymbol{b} = \frac{1}{2}\left(\boldsymbol{a}\,\boldsymbol{b} + \boldsymbol{b}\,\boldsymbol{a}\right)$$

$$\boldsymbol{a}\times\boldsymbol{b} = \frac{1}{2i}\left(\boldsymbol{a}\,\boldsymbol{b} - \boldsymbol{b}\,\boldsymbol{a}\right)$$

$\boldsymbol{a}\cdot\boldsymbol{b} = 0 \longrightarrow \boldsymbol{a} \perp \boldsymbol{b} \longrightarrow \boldsymbol{a}\,\boldsymbol{b} = -\boldsymbol{a}\,\boldsymbol{b}$

$\boldsymbol{a}\times\boldsymbol{b} = 0 \longrightarrow \boldsymbol{a} \parallel \boldsymbol{b} \longrightarrow \boldsymbol{a}\,\boldsymbol{b} = \boldsymbol{a}\,\boldsymbol{b}$

Vector Rotation

$$e^{i\boldsymbol{e}_3\theta} = \cos\theta + i\,\boldsymbol{e}_3 \sin\theta$$

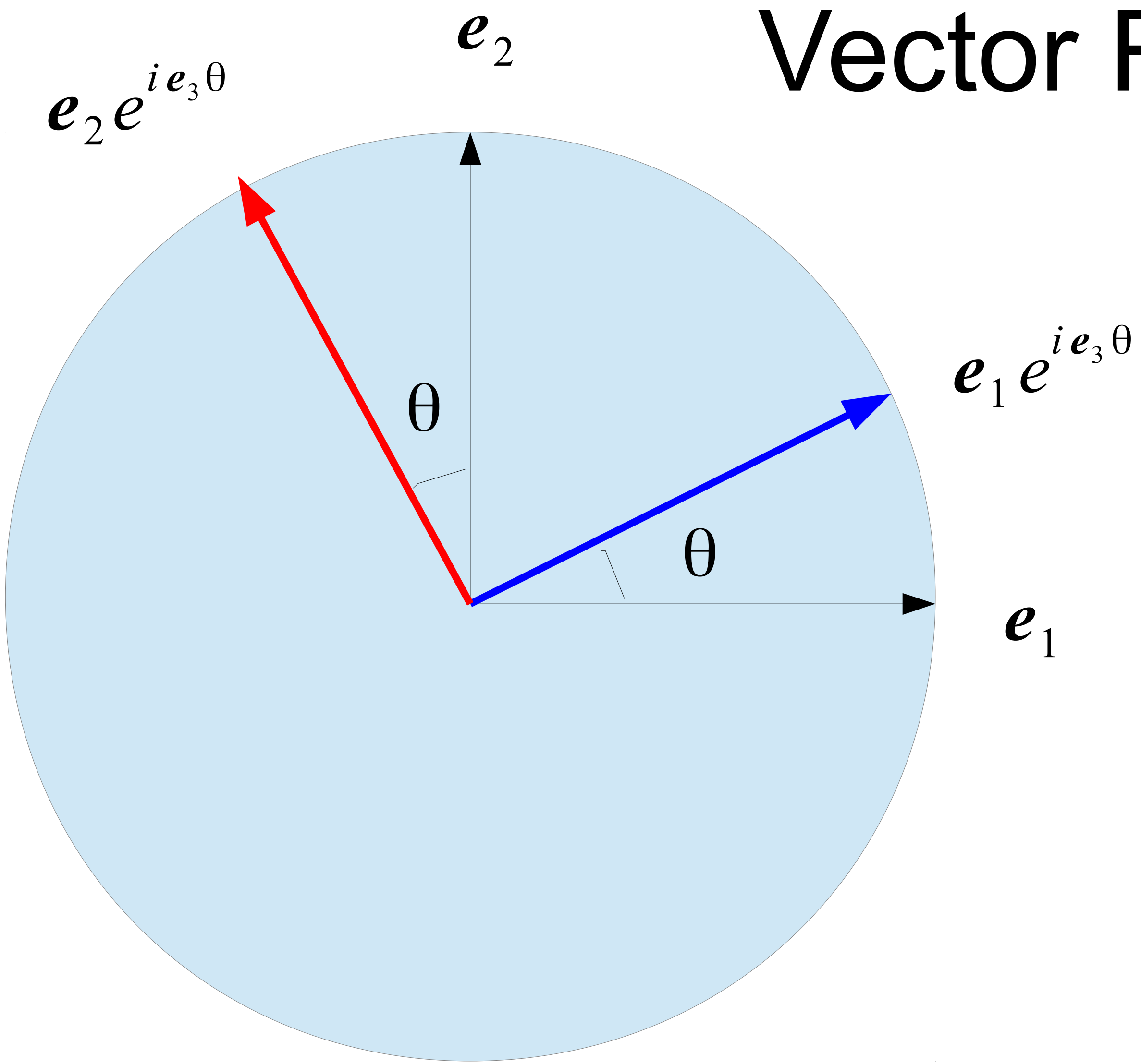
$$\boldsymbol{e}_1\, e^{i\boldsymbol{e}_3\theta} = \boldsymbol{e}_1\left(\cos\theta + i\,\boldsymbol{e}_3 \sin\theta\right) = \boldsymbol{e}_1 \cos\theta + \boldsymbol{e}_2 \sin\theta$$

$$\boldsymbol{e}_2\, e^{i\boldsymbol{e}_3\theta} = \boldsymbol{e}_2\left(\cos\theta + i\,\boldsymbol{e}_3 \sin\theta\right) = \boldsymbol{e}_1 \cos\theta - \boldsymbol{e}_1 \sin\theta$$

$$\boldsymbol{e}_1 i \boldsymbol{e}_3 = \boldsymbol{e}_1 \boldsymbol{e}_1 \boldsymbol{e}_2 = \boldsymbol{e}_2$$

$$\boldsymbol{e}_2 i \boldsymbol{e}_3 = \boldsymbol{e}_2 \boldsymbol{e}_1 \boldsymbol{e}_2 = -\boldsymbol{e}_1$$

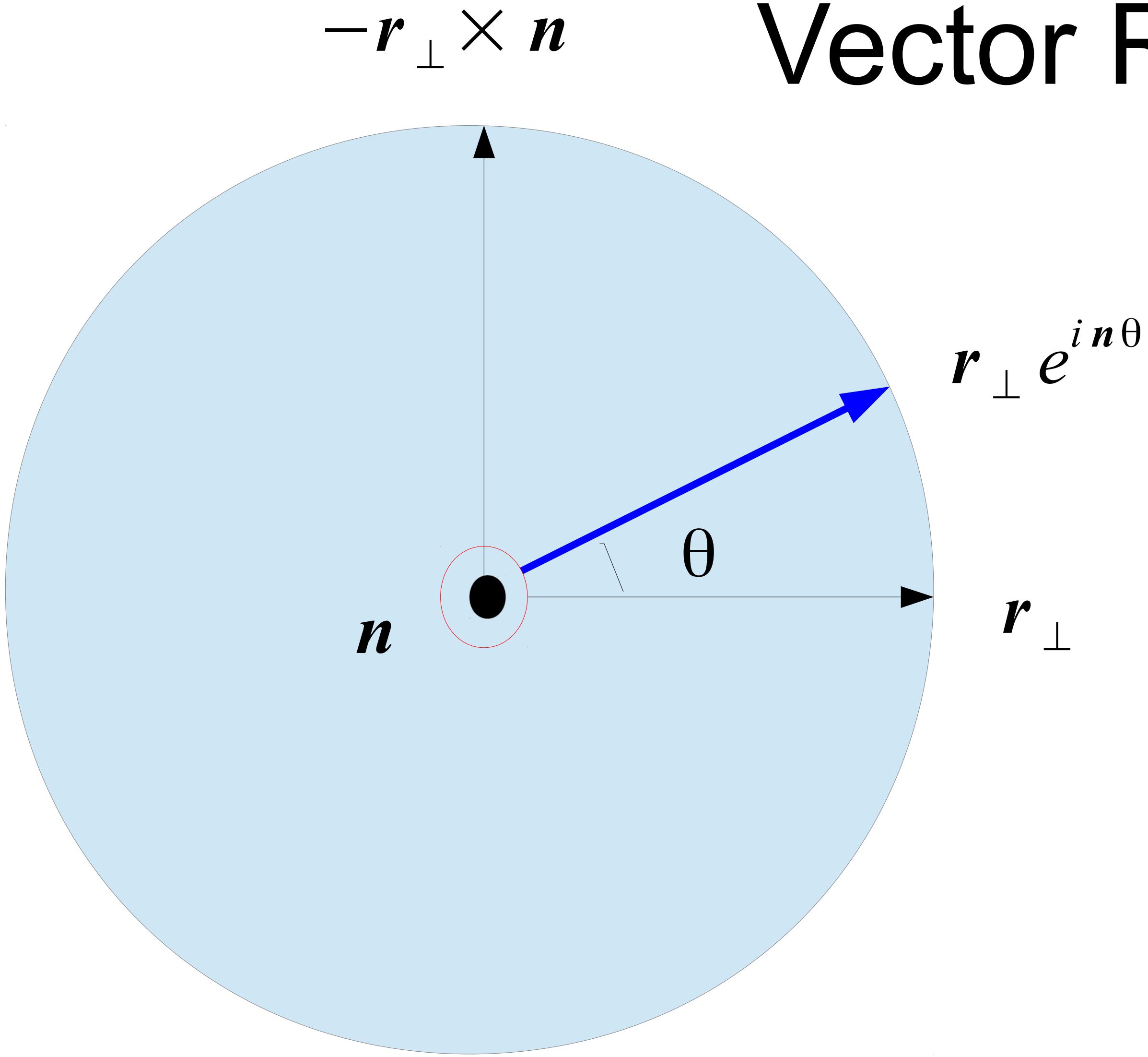
Vector Rotation in 2D



$$\boldsymbol{e}_1\, e^{i\boldsymbol{e}_3\theta} = \boldsymbol{e}_1 \cos\theta + \boldsymbol{e}_2 \sin\theta = e^{-i\boldsymbol{e}_3\theta}\, \boldsymbol{e}_1$$

$$\boldsymbol{e}_2\, e^{i\boldsymbol{e}_3\theta} = \boldsymbol{e}_2 \cos\theta - \boldsymbol{e}_1 \sin\theta = e^{-i\boldsymbol{e}_3\theta}\, \boldsymbol{e}_2$$

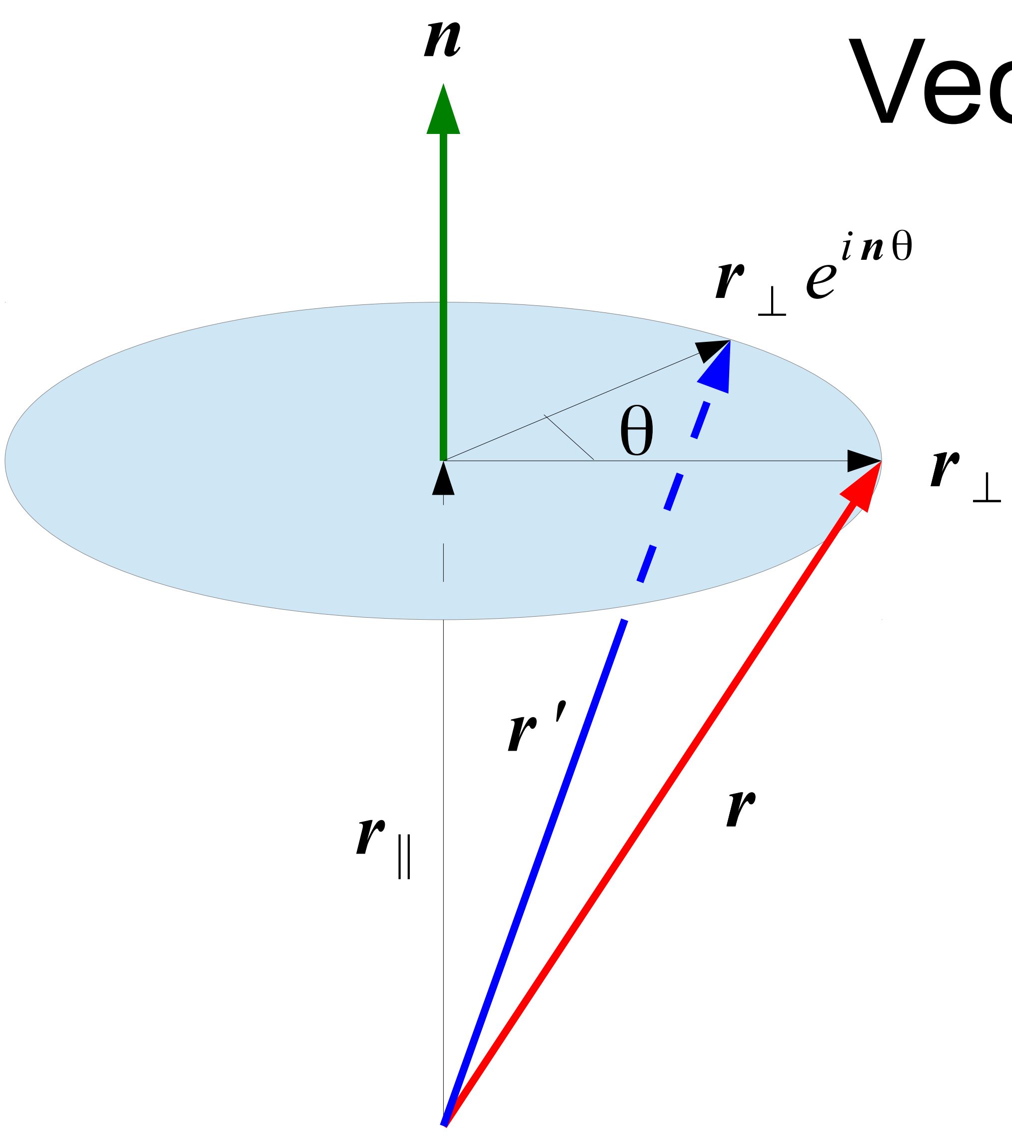
Vector Rotation in 2D



$$\boldsymbol{r}_\perp e^{i\boldsymbol{n}\theta} = \boldsymbol{r}_\perp\left(\cos\theta + i\boldsymbol{n}\sin\theta\right); \quad \boldsymbol{r}_\perp \cdot \boldsymbol{n} = 0$$

$$= \boldsymbol{r}_\perp \cos\theta - \boldsymbol{r}_\perp \times \boldsymbol{n}\sin\theta$$

Vector Rotation in 3D



$$\boldsymbol{r}_{\parallel}\, e^{i\boldsymbol{n}\theta} = e^{i\boldsymbol{n}\theta}\, \boldsymbol{r}_{\parallel}$$

$$\boldsymbol{r}_{\perp}\, e^{i\boldsymbol{n}\theta} = e^{-i\boldsymbol{n}\theta}\, \boldsymbol{r}_{\perp}$$

$$\boldsymbol{r}' = e^{-i\boldsymbol{n}\theta/2}\, \boldsymbol{r}\, e^{i\boldsymbol{n}\theta/2}$$

$$= e^{-i\boldsymbol{n}\theta/2} \left(\boldsymbol{r}_{\parallel} + \boldsymbol{r}_{\perp}\right) e^{i\boldsymbol{n}\theta/2}$$

$$\boxed{\boldsymbol{r}' = \boldsymbol{r}_{\parallel} + \boldsymbol{r}_{\perp}\, e^{i\boldsymbol{n}\theta}}$$

The vector **r'** is the vector **r** rotated counterclockwise about **n** by an angle θ

Magnetic Force

$$\boldsymbol{F} = m\frac{d\,\boldsymbol{v}}{d\,t} = q\,\boldsymbol{v}\times\boldsymbol{B}; \qquad \boldsymbol{v} = \boldsymbol{v}_{\parallel} + \boldsymbol{v}_{\perp}$$

$$m\frac{d\,\boldsymbol{v}_{\parallel}}{d\,t} = 0$$

(Parallel to **B**)

$$m\frac{d\,\boldsymbol{v}_{\perp}}{d\,t} = q\,\boldsymbol{v}_{\perp}\times\boldsymbol{B};$$

(Perpendicular to **B**)

Motion in Magnetic Field

$$m \frac{d\,\boldsymbol{v}_{\parallel}}{d\,t} = 0$$

$$\boldsymbol{v}_{\parallel} = \boldsymbol{v}_{0\,\parallel}$$ (constant velocity parallel to **B**)

$$\boldsymbol{r}_{\parallel} = \boldsymbol{v}_{0\,\parallel}\, t + \boldsymbol{r}_{0\,\parallel}$$ (uniform linear motion parallel to **B**)

Motion in Magnetic Force

$$m\frac{d\boldsymbol{v}_\perp}{dt} = q\boldsymbol{v}_\perp \times \boldsymbol{B};$$

$$m\frac{d\boldsymbol{v}_\perp}{dt} = -iq\boldsymbol{v}_\perp \boldsymbol{B};$$

$$\boldsymbol{v}_\perp \boldsymbol{B} = \underbrace{\boldsymbol{v}_\perp \cdot \boldsymbol{B}}_{0} + i\boldsymbol{v}_\perp \times \boldsymbol{B}$$

$$= i\left(\boldsymbol{v}_\perp \times \boldsymbol{B}\right)$$

$$\boxed{\boldsymbol{v}_\perp = \boldsymbol{v}_{\perp 0} e^{i\omega t};} \quad \boldsymbol{v}_{\perp 0} \perp \boldsymbol{\omega}$$

$$\frac{d}{dt}\boldsymbol{v}_\perp = \boldsymbol{v}_{\perp 0}(i\boldsymbol{\omega})e^{i\omega t};$$

$$im\boldsymbol{v}_{\perp 0}\boldsymbol{\omega} e^{i\omega t} = -iq\boldsymbol{v}_{\perp 0}e^{i\omega t}\boldsymbol{B}; \quad \boldsymbol{B} \parallel \boldsymbol{\omega}$$

$$\boxed{\boldsymbol{\omega} = -\frac{q}{m}\boldsymbol{B}} \quad \text{(gyrofrequency)}$$

Motion in Magnetic Force

$$\boldsymbol{\omega} = -\frac{q}{m}\boldsymbol{B}$$

$$\boldsymbol{r} = \boldsymbol{r}_{0\parallel} + \boldsymbol{v}_{0\parallel} t + \boldsymbol{r}_{0\perp} e^{i\omega t}$$

- $\boldsymbol{r}_{0\perp} e^{i\omega t}$ → Uniform circular motion of the vector about the magnetic field
- $\boldsymbol{v}_{0\parallel} t$ → Linear motion with constant velocity
- $\boldsymbol{r}_{0\parallel}$ → Initial position

Maxwell's Equation

$$\frac{\partial \hat{E}}{\partial \hat{r}} = \zeta \hat{j}^{+}$$

$$\frac{\partial}{\partial \hat{r}} = \frac{1}{c}\frac{\partial}{\partial t} + \nabla$$

Spacetime derivative operator

$$\hat{E} = \boldsymbol{E} + \zeta \boldsymbol{H}$$

Electromagnetic field

$$\hat{j}^{+} = \rho c - \boldsymbol{j}$$

Spatial inverse of spacetime current density

The spacetime derivative of the electromagnetic field is proportional to the spacetime current density

Maxwell's Equation

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \nabla\right)(\boldsymbol{E} + i\zeta\boldsymbol{H}) = \zeta(\rho c - \boldsymbol{j})$$

Note:

$$\nabla\boldsymbol{E} = \nabla\cdot\boldsymbol{E} + i(\nabla\times\boldsymbol{E})$$

$$\nabla\boldsymbol{H} = \nabla\cdot\boldsymbol{H} + i(\nabla\times\boldsymbol{H})$$

$$\zeta = (\mu_0/\epsilon_0)^{1/2}$$

$$c = (\mu_0\epsilon_0)^{-1/2}$$

Maxwell's Equations

0-vector: $$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$$ Gauss's Law

1-vector: $$\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} - \zeta \nabla \times \boldsymbol{H} = -\zeta \boldsymbol{j}$$ Ampere's law

2-vector: $$i\left[\frac{\zeta}{c}\frac{\partial \boldsymbol{H}}{\partial t} + \nabla \times \boldsymbol{E} = 0\right]$$ Faraday's law

3-vector: $$i\left[\zeta \nabla \cdot \boldsymbol{H} = 0\right]$$ Magnetic flux continuity law

Magnetic field of line current

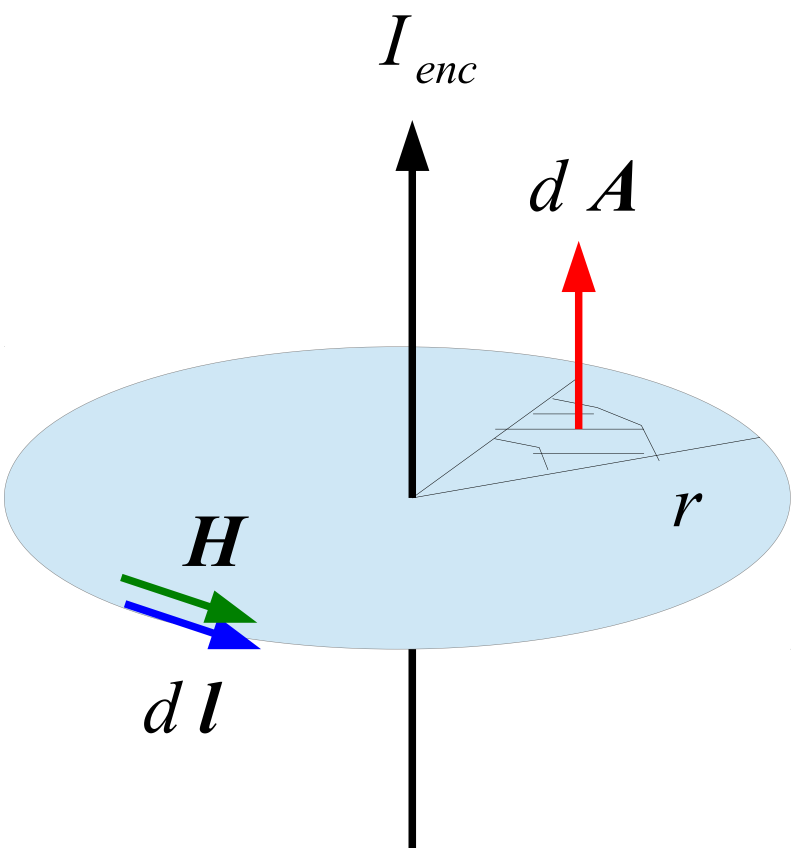
$$\nabla \times \boldsymbol{H} = \boldsymbol{j}$$

$$\oint_S (\nabla \times \boldsymbol{H}) \cdot d\boldsymbol{A} = \oint_S \boldsymbol{j} \cdot d\boldsymbol{A}$$

$$\oint_C \boldsymbol{H} \cdot d\boldsymbol{l} = I_{enc}$$

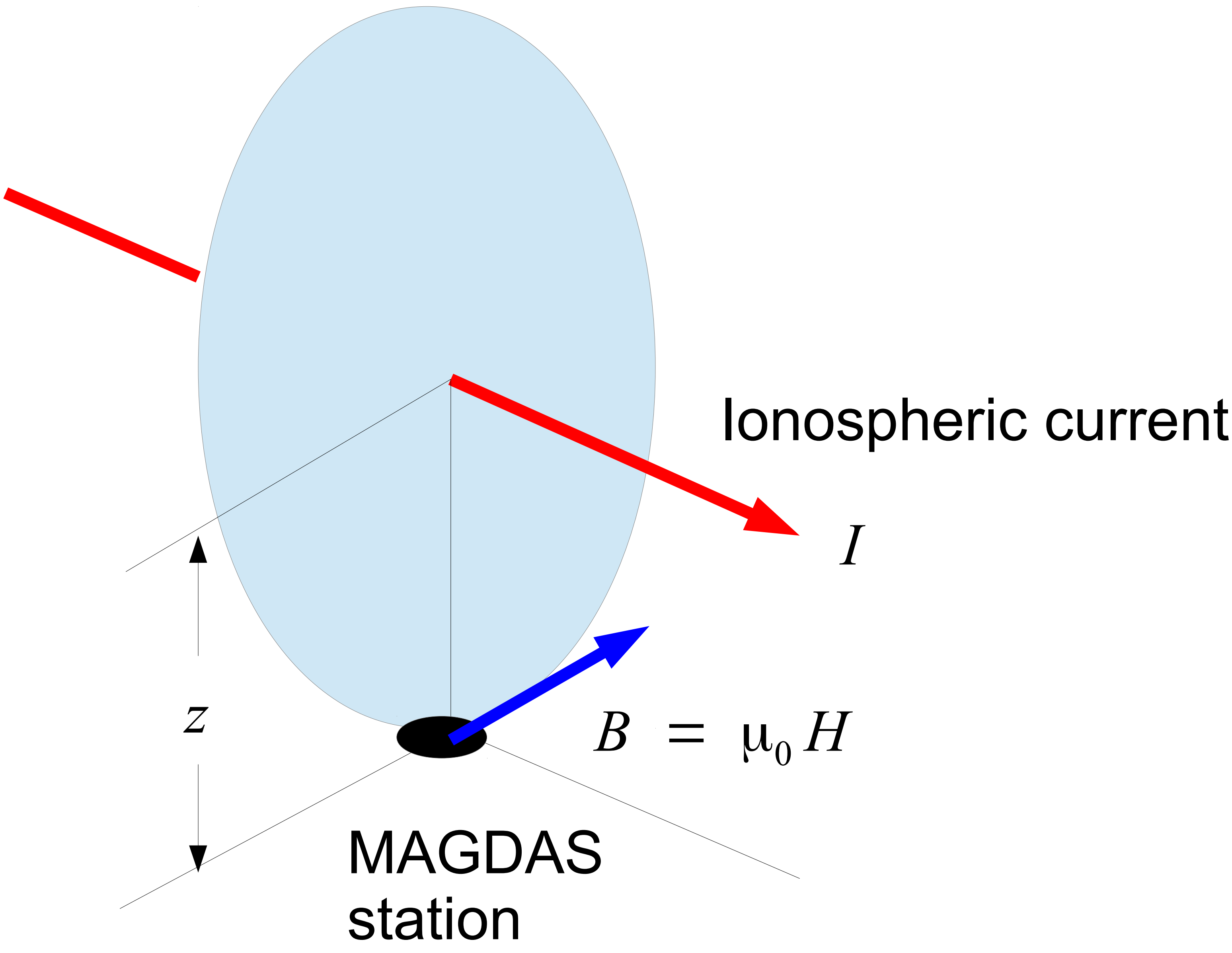
$$H_\theta (2\pi r) = I_{enc}$$

$$H_\theta = \frac{I_{enc}}{2\pi r}$$

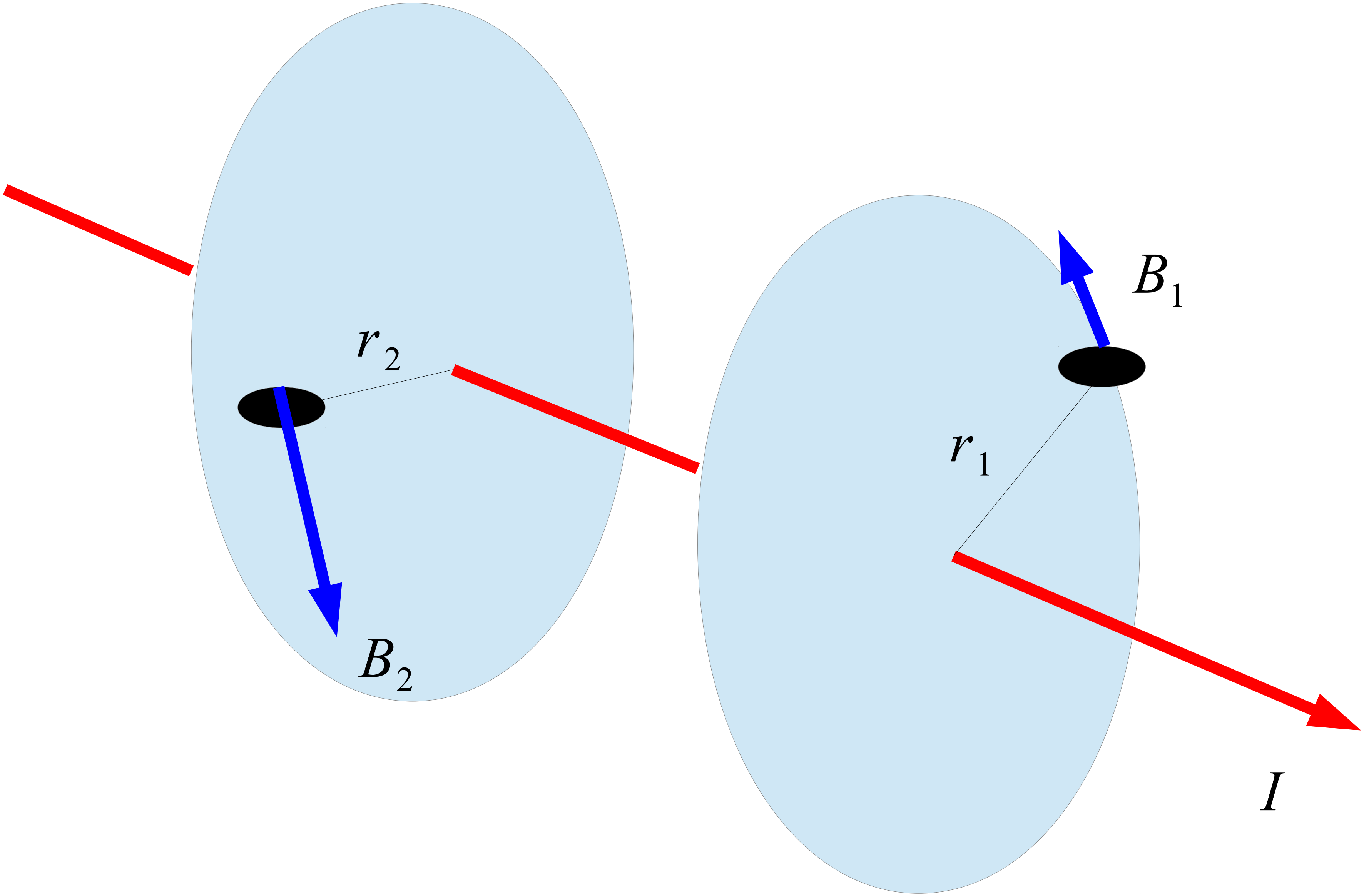


$$Note: \quad \boldsymbol{B} = \mu_0 \boldsymbol{H}$$

Ionospheric currents



Lithospheric currents



Wave Equations

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)(\boldsymbol{E} + i\zeta\boldsymbol{H}) \;=\; \zeta\left(\frac{1}{c}\frac{\partial}{\partial t} - \nabla\right)(\rho c - \boldsymbol{j})$$

0-vector: $$0 \;=\; \frac{\partial\rho}{\partial t} - \nabla\cdot\boldsymbol{j}$$

1-vector: $$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\boldsymbol{E} \;=\; -\mu_0\frac{\partial\boldsymbol{j}}{\partial t} - \frac{1}{\epsilon_0}\nabla\rho$$

2-vector: $$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\zeta\boldsymbol{H} \;=\; -\zeta\nabla\times\boldsymbol{j}$$

Wave Equations

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)(\boldsymbol{E} + i\zeta\boldsymbol{H}) = 0$$

1-vector:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\boldsymbol{E} = 0$$

2-vector:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\zeta\boldsymbol{H} = 0$$

Electromagnetic Wave

$$\boldsymbol{E} + i\zeta \boldsymbol{H} = (\boldsymbol{E}_0 + i\zeta \boldsymbol{H}_0) e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}$$

$$= (\boldsymbol{E}_0 + i\zeta \boldsymbol{H}_0)(\cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}) + i\sin(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}))$$

1-vector: $$\boldsymbol{E} = \boldsymbol{E}_0 \cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}) - \zeta \boldsymbol{H}_0 \sin(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})$$

2-vector: $$\zeta \boldsymbol{H} = \zeta \boldsymbol{H}_0 \cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}) + \zeta \boldsymbol{E}_0 \sin(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})$$

Wave Condition

$$0 = \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\left[\left(\boldsymbol{E}_{\mathbf{0}} + i\zeta\boldsymbol{H}_{\mathbf{0}}\right)e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}\right]$$

$$0 = \left(\frac{\omega^2}{c^2} - \boldsymbol{k}^2\right)\left(\boldsymbol{E}_{\mathbf{0}} + i\zeta\boldsymbol{H}_{\mathbf{0}}\right)e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}$$

$$\frac{\omega}{c} = k$$

Eigenvalue Equation

$$0 = \left(\frac{1}{c}\frac{\partial}{\partial t} + \nabla\right)\left(\boldsymbol{E} + i\zeta\boldsymbol{H}\right)$$

$$= \left(\frac{1}{c}\frac{\partial}{\partial t} + \nabla\right)\left[\left(\boldsymbol{E}_{\mathbf{0}} + i\zeta\boldsymbol{H}_{\mathbf{0}}\right)e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}\right]$$

$$= i\left(\frac{\omega}{c} - \boldsymbol{k}\right)\left[\left(\boldsymbol{E}_{\mathbf{0}} + i\zeta\boldsymbol{H}_{\mathbf{0}}\right)e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}\right]$$

$$\boxed{0 = \left(\frac{\omega}{c} - \boldsymbol{k}\right)\left(\boldsymbol{E}_{\mathbf{0}} + i\zeta\boldsymbol{H}_{\mathbf{0}}\right)}$$

Orthonormality Relations

$$0 = \left(\frac{\omega}{c} - \boldsymbol{k}\right)\left(\boldsymbol{E}_0 + i\zeta \boldsymbol{H}_0\right)$$

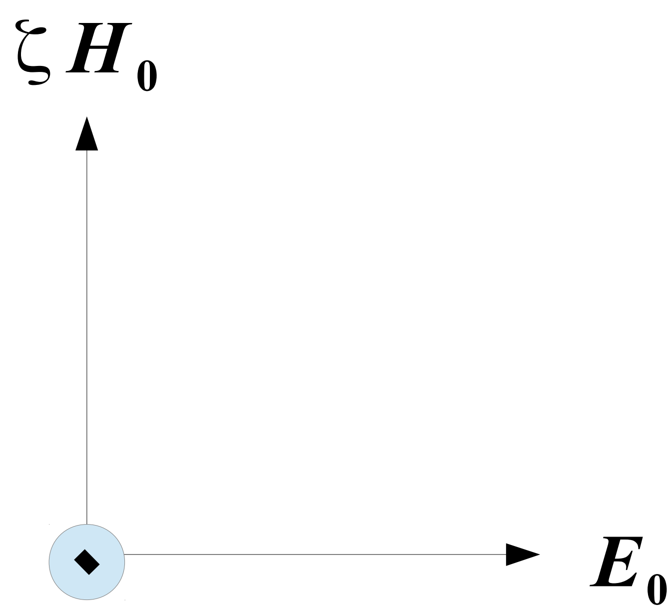
0-vector: $0 = \boldsymbol{k} \cdot \boldsymbol{E}_0$

1-vector: $0 = \frac{\omega}{c} \boldsymbol{E}_0 + \zeta \boldsymbol{k} \times \boldsymbol{H}_0$

2-vector: $0 = i\left[\zeta \frac{\omega}{c} \boldsymbol{H}_0 - \boldsymbol{k} \times \boldsymbol{E}_0\right]$

3-vector: $0 = i\zeta \boldsymbol{k} \cdot \boldsymbol{H}_0$

Electromagnetic Field Amplitudes



$$e_k = \frac{k}{k}$$

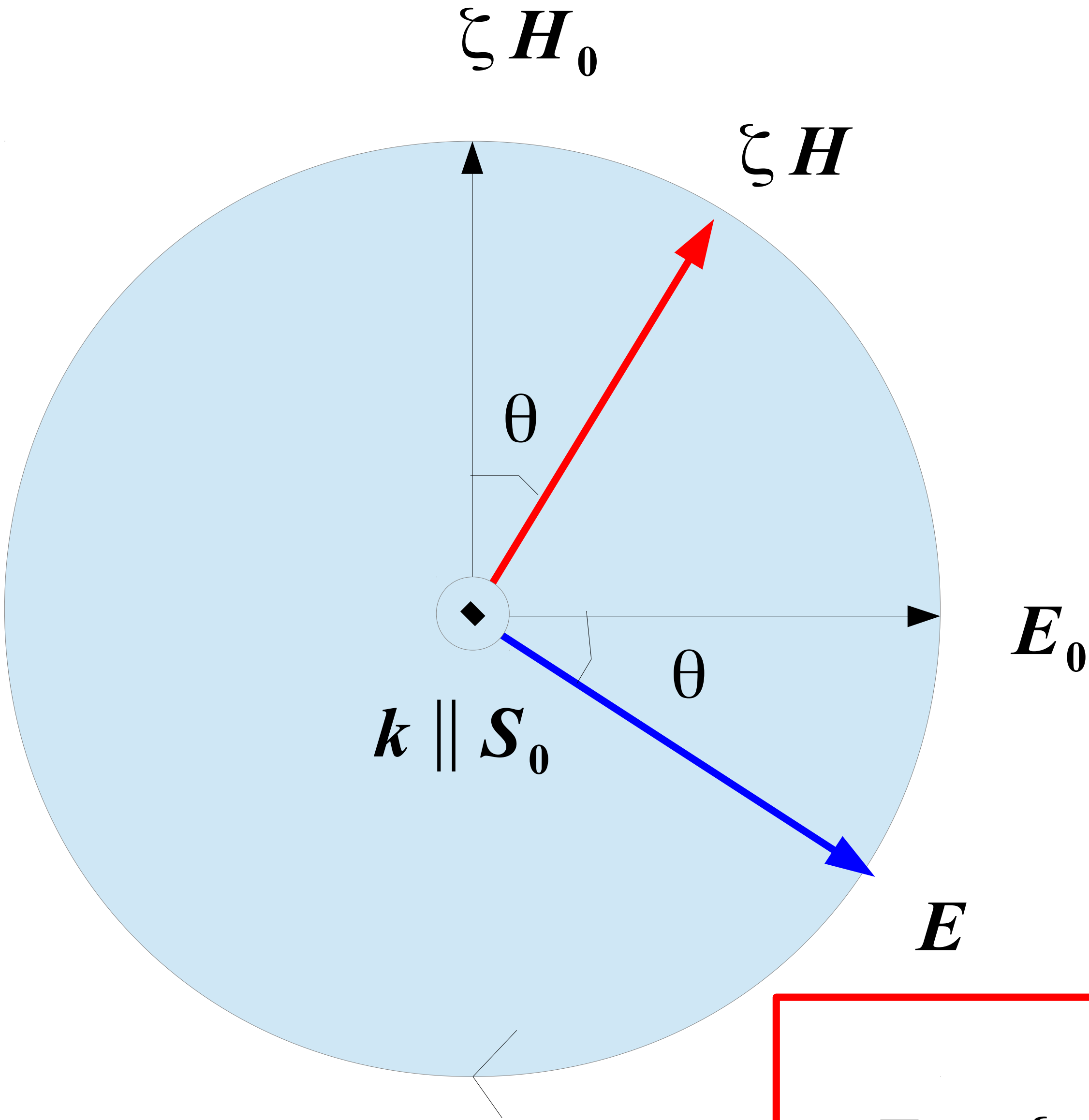
$$|E_0| = \zeta |H_0|$$

$$E_0 = \zeta H_0 \times e_k$$

$$\zeta H_0 = -E_0 \times e_k$$

$$S_0 = E_0 \times H_0$$

Electromagnetic Wave



$$\theta = \omega t - \boldsymbol{k}\cdot\boldsymbol{r}$$

$$\boldsymbol{E} + i\zeta \boldsymbol{H} = \left(\boldsymbol{E}_0 + i\zeta \boldsymbol{H}_0\right) e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}$$

Electromagnetic Wave

$$\boldsymbol{E} = \boldsymbol{E}_0 \cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}) - \zeta \boldsymbol{H}_0 \sin(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})$$

$$\zeta \boldsymbol{H}_0 = -\boldsymbol{E}_0 \times \boldsymbol{e}_k$$

$$\boldsymbol{E} = \boldsymbol{E}_0 \cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{r}) + \boldsymbol{E}_0 \times \boldsymbol{e}_k \sin(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})$$

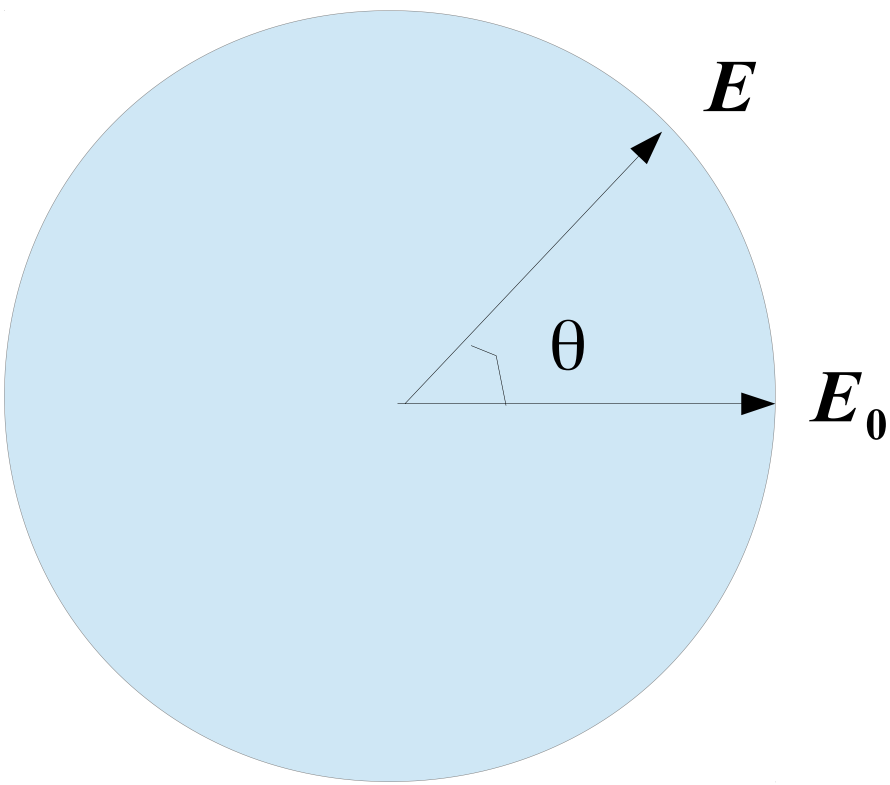
$$\boldsymbol{r}_\perp e^{i\boldsymbol{n}\theta} = \boldsymbol{r}_\perp \cos\theta - \boldsymbol{r}_\perp \times \boldsymbol{n} \sin\theta$$

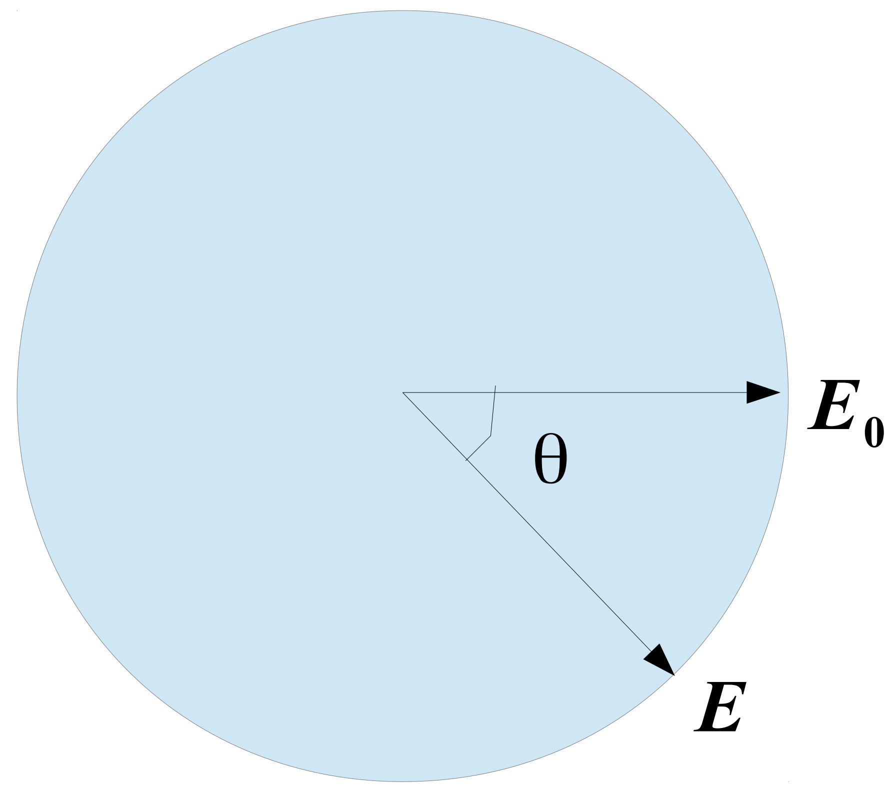
$$\boxed{\boldsymbol{E} = \boldsymbol{E}_0 e^{-i\boldsymbol{e}_k(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}}$$

Circularly polarized EM wave

$$\boldsymbol{E} = \boldsymbol{E_0} e^{i \boldsymbol{e}_k (\omega t - \boldsymbol{k} \cdot \boldsymbol{r})}$$

$$\boldsymbol{E} = \boldsymbol{E_0} e^{-i \boldsymbol{e}_k (\omega t - \boldsymbol{k} \cdot \boldsymbol{r})}$$

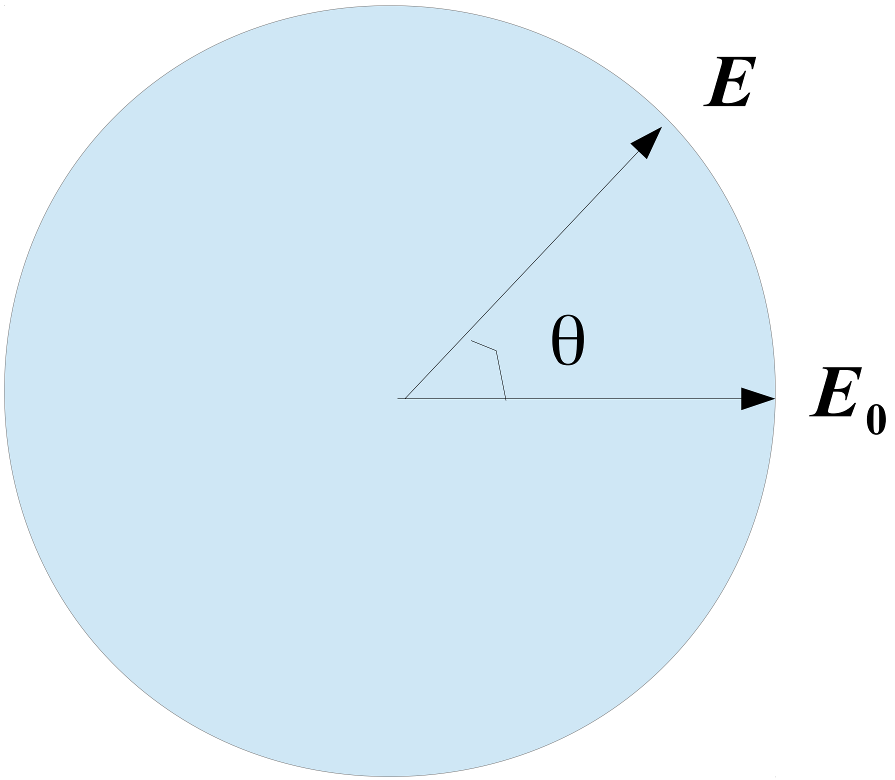




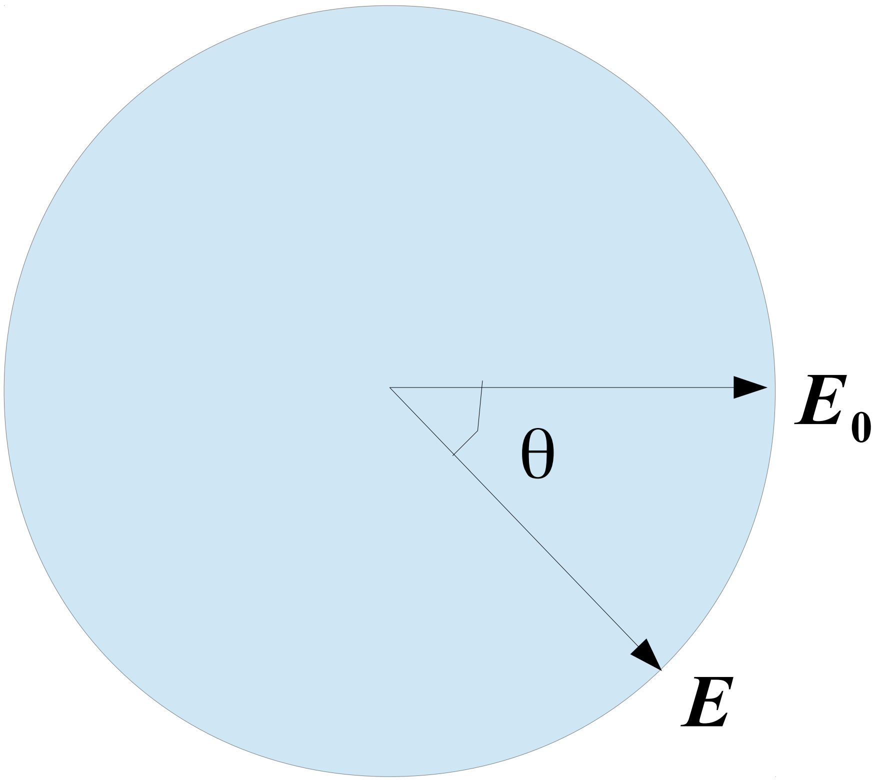
$$\theta = \omega t - \boldsymbol{k} \cdot \boldsymbol{r}$$

Circularly polarized EM wave

$$\boldsymbol{E} = \boldsymbol{E_0} e^{i \boldsymbol{e}_k (\omega t - \boldsymbol{k} \cdot \boldsymbol{r})}$$

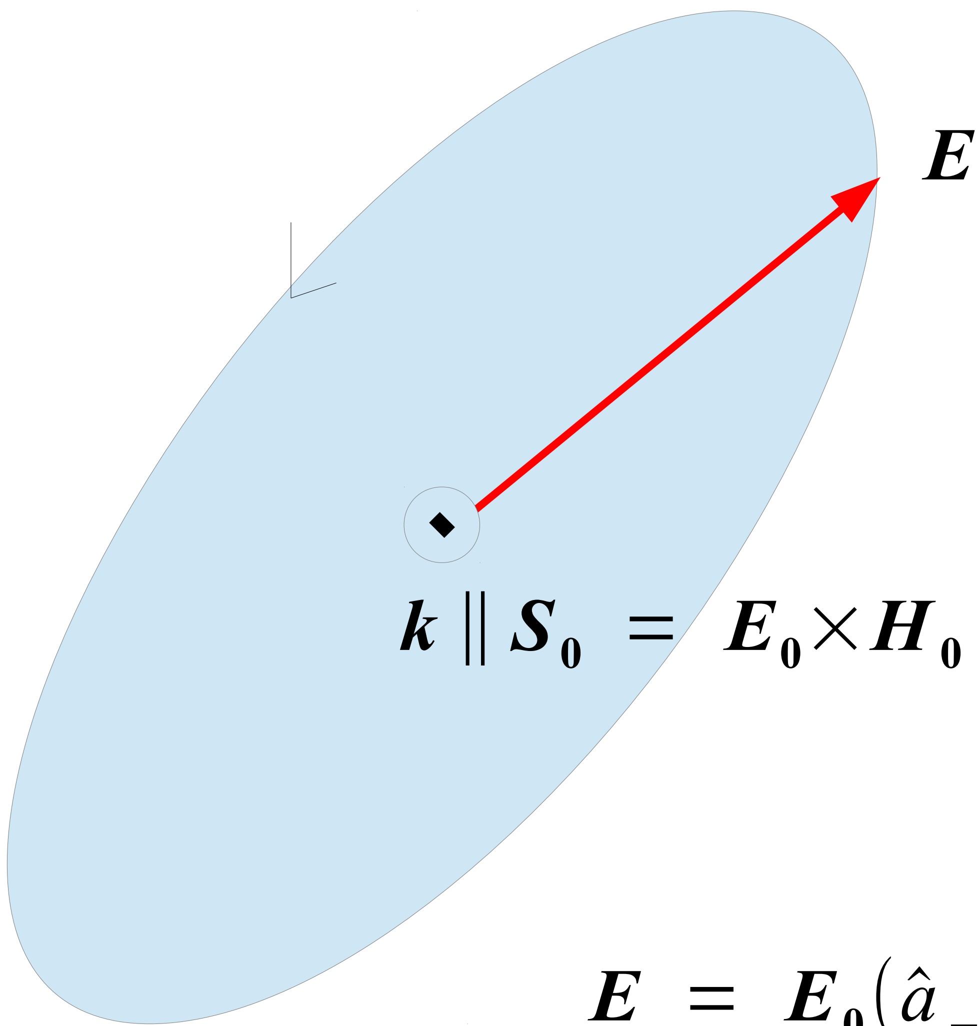


$$\boldsymbol{E} = \boldsymbol{E_0} e^{-i \boldsymbol{e}_k (\omega t - \boldsymbol{k} \cdot \boldsymbol{r})}$$



$$\theta = \omega t - \boldsymbol{k} \cdot \boldsymbol{r}$$

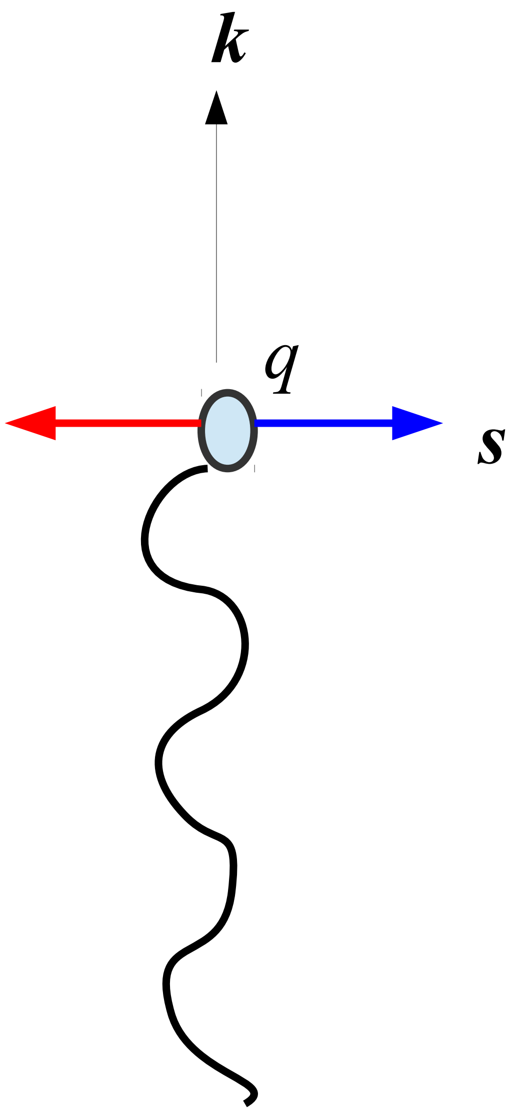
Elliptically polarized EM wave



$$\boldsymbol{k} \parallel \boldsymbol{S}_0 = \boldsymbol{E}_0 \times \boldsymbol{H}_0$$

$$\boldsymbol{E} = \boldsymbol{E}_0 \left(\hat{a}_- e^{-i \boldsymbol{e}_k (\omega t - \boldsymbol{k} \cdot \boldsymbol{r})} + \hat{a}_+ e^{i \boldsymbol{e}_k (\omega t - \boldsymbol{k} \cdot \boldsymbol{r})} \right)$$

Charge motion due to EM wave



Equation of Motion:

$$\boldsymbol{F} = q\boldsymbol{E} = m\frac{\partial^2 \boldsymbol{s}}{\partial t^2}$$

Assumed solution:

$$\boldsymbol{s} = \gamma \boldsymbol{E} = \gamma \boldsymbol{E_0} e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}$$

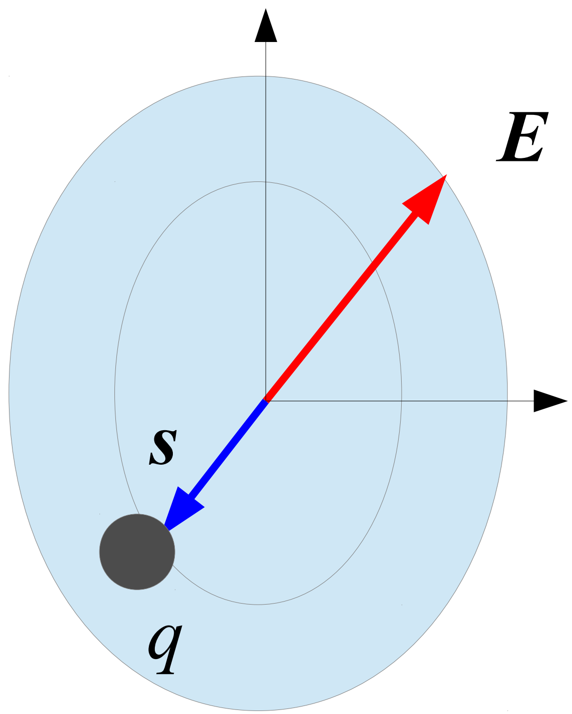
$$\frac{\partial^2 \boldsymbol{s}}{\partial t^2} = \gamma \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = -\omega^2 \gamma \boldsymbol{E_0} e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}$$

$$= -\gamma \omega^2 \boldsymbol{E}$$

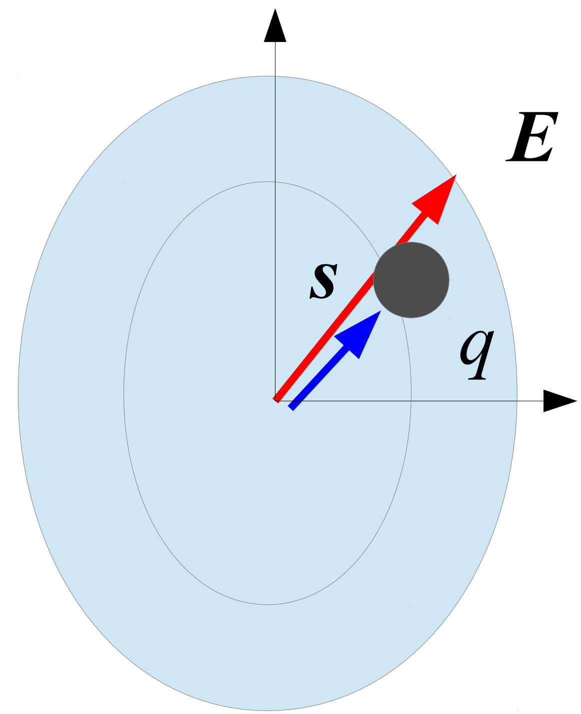
Charge motion in an EM wave

$$\boldsymbol{F} = q\boldsymbol{E} = -m\gamma\omega^2\boldsymbol{E} \longrightarrow \gamma = -\frac{q}{m\omega^2}$$

$$\boldsymbol{s} = \gamma\boldsymbol{E} = -\frac{q}{m\omega^2}\boldsymbol{E}$$



$q > 0$ (proton)



$q < 0$ (electron)

EM wave in ionosphere

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{j}}{\partial t} - \frac{1}{\epsilon_0}\nabla \rho$$

$$\nabla \rho \approx 0$$

Uniform charge density

$$\boldsymbol{j} = N q \boldsymbol{v} = N q \frac{\partial \boldsymbol{s}}{\partial t}$$

$$\boldsymbol{s} = -\frac{q}{m\omega^2}\boldsymbol{E}$$

$$\frac{\partial \boldsymbol{j}}{\partial t} = N q \frac{\partial^2 \boldsymbol{s}}{\partial t^2} = -\frac{Nq^2}{m\omega^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

EM Wave in ionosphere

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\boldsymbol{E} = \mu_0 \frac{Nq^2}{m\omega^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2}; \qquad c^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$\left(\frac{n^2}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\boldsymbol{E} = 0$$

$$n = \sqrt{1 - \frac{Nq^2}{\epsilon_0 m \omega^2}}$$

$$\omega = 2\pi f$$

$$n = \sqrt{1 - \frac{Nq^2}{4\pi^2 \epsilon_0 m f^2}}$$

Plasma refractive index

$$n = \sqrt{1 - \frac{Nq^2}{4\pi^2 \epsilon_0 m f^2}}$$

$$q = 1.6 \times 10^{-19}\ C$$

$$m_e = 9.11 \times 10^{-31}\ kg$$

$$\epsilon_0 = 8.85 \times 10^{-12}\ F/m$$

$$n = \sqrt{1 - \frac{81\,N}{f^2}}$$

Phase Modulation

Perfectly sinusoidal signal = no information transmitted

$$E = \cos(\omega t - kz)$$

phase

angular frequency

DSBSC = double side band suppressed carrier (one method)

$$E = \cos(\omega_1 t - k_1 z) + \cos(\omega_2 t - k_2 z)$$

$$= \cos(\omega_0 t - k_0 z)\cos(\Delta\omega t - \Delta k z)$$

Phase Modulation

$$E = \cos\left(\omega_0 t - k_0 z\right) \cos\left(\Delta\omega t - \Delta k z\right)$$

carrier — $\cos\left(\omega_0 t - k_0 z\right)$

modulation — $\cos\left(\Delta\omega t - \Delta k z\right)$

$$\omega_0 = \frac{1}{2}\left(\omega_1 + \omega_2\right)$$

$$k_0 = \frac{1}{2}\left(k_1 + k_2\right)$$

$$\Delta\omega = \frac{1}{2}\left(\omega_1 - \omega_2\right)$$

$$\Delta k = \frac{1}{2}\left(k_1 + k_2\right)$$

Group and Phase Velocity

$$k = \frac{\omega}{c} n$$

$$n = \sqrt{1 - \frac{Nq^2}{\epsilon_0 m \omega^2}}$$

$$v_{group} = \frac{\partial z}{\partial t} = \frac{\partial k}{\partial \omega} = nc$$

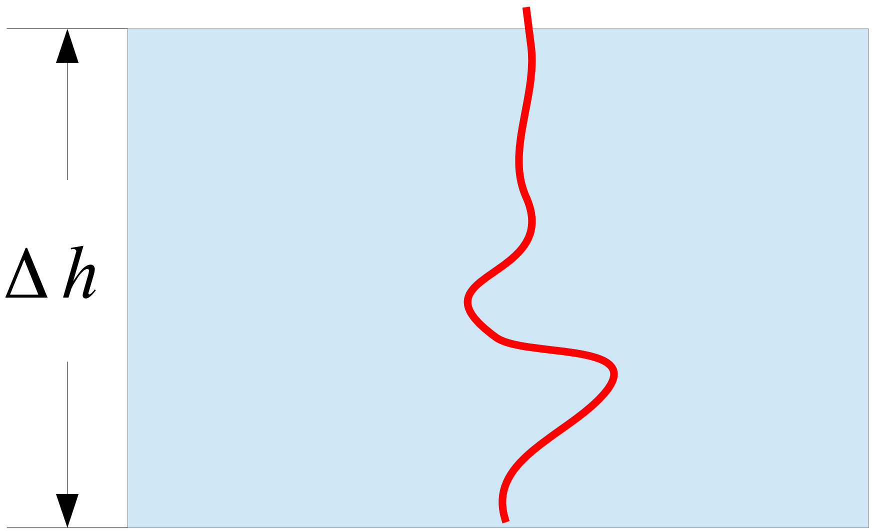
$$v_{phase} = \frac{\partial \omega}{\partial k} = \frac{c}{n}$$

$$v_{group} v_{phase} = c^2$$

Wave pulse propagation in ionosphere

$$v_{group} = nc = c\sqrt{1-\frac{81N}{f^2}}$$

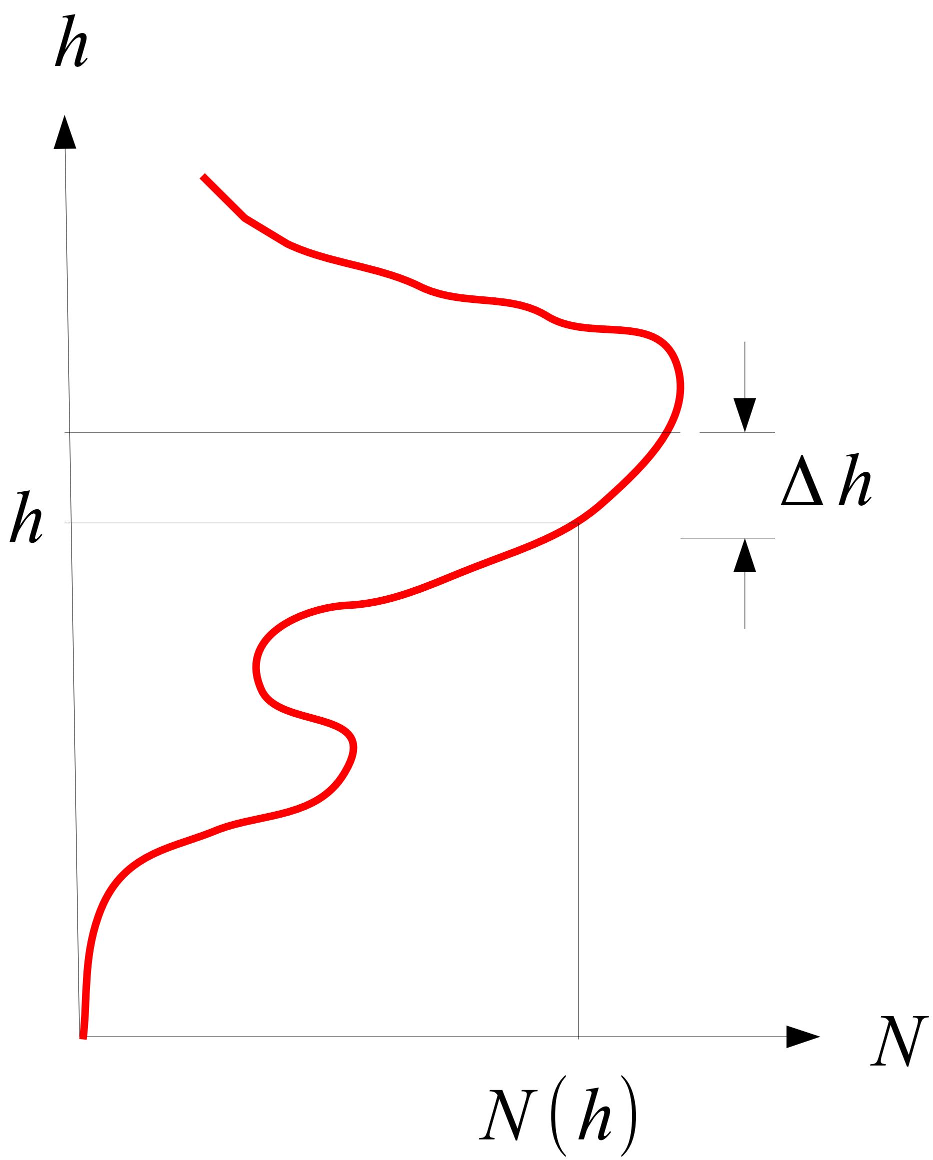
$$\Delta t = \frac{\Delta h}{v} = \frac{\Delta h}{c\sqrt{1-\frac{81N}{f^2}}}$$



Virtual thickness

$$c\Delta t = \frac{\Delta h}{\sqrt{1-\frac{81N}{f^2}}}$$

Ionogram construction

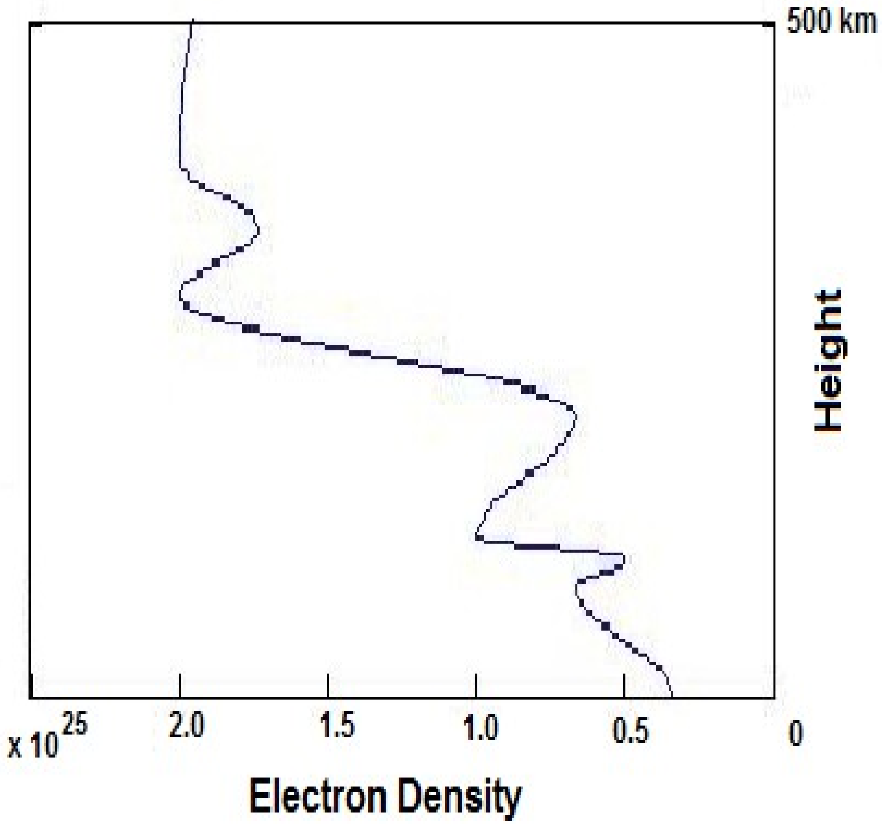


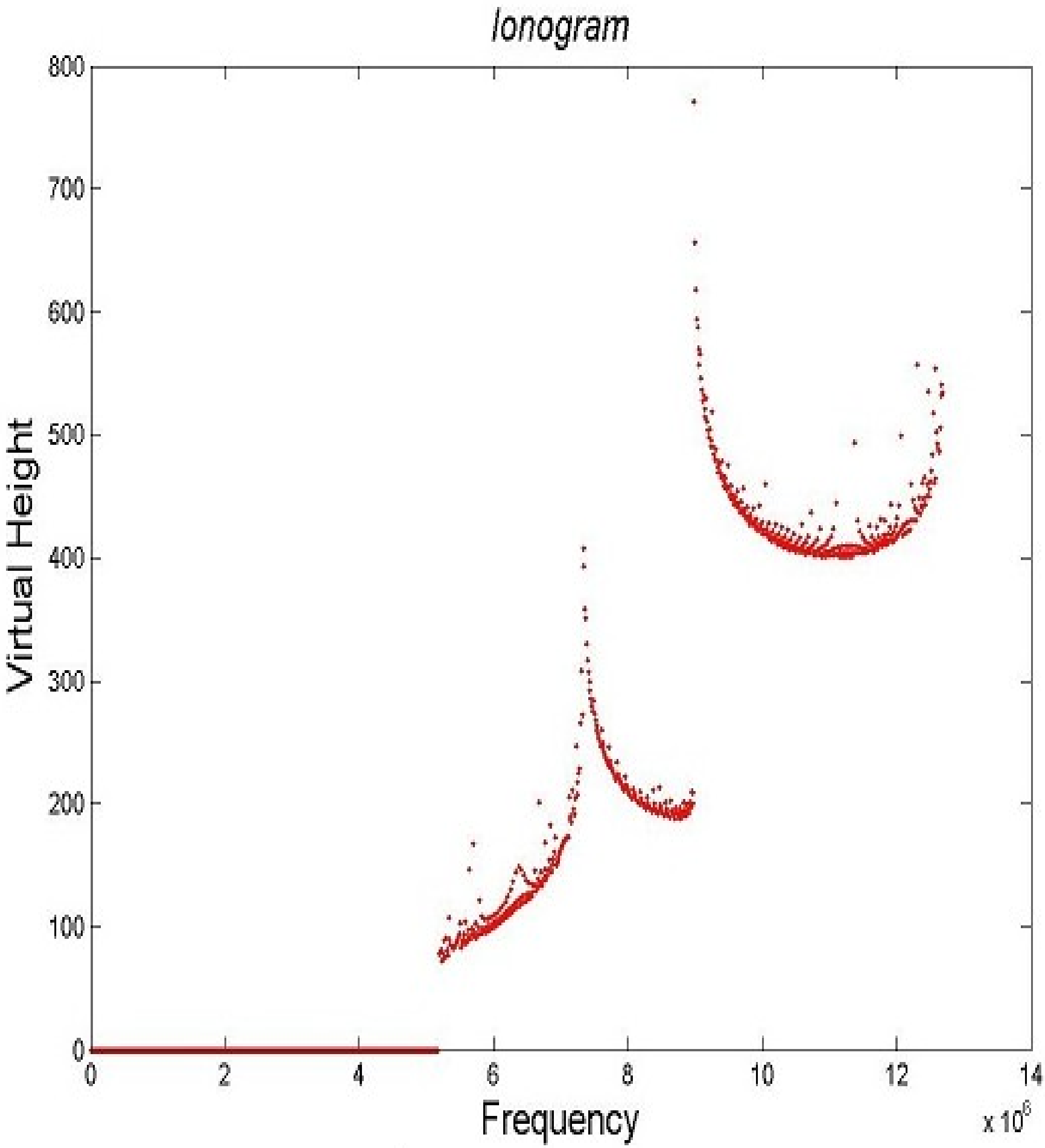
$$\sum_{j=1}^{j_{max}} c(\Delta t)_j = \sum_{j=1}^{j_{max}} \frac{(\Delta h_j)}{\sqrt{1-\frac{81N}{f^2}}}$$

$$h' = \int_0^{t_{max}} c\,dt = \int_0^{h_{max}} \frac{dh}{\sqrt{1-\frac{81N}{f^2}}}$$

$$N = N(h)$$

Electron density profile and ionogram





Future Works

1. Simulation of ionograms in geomagnetic field with multiple inter-layer reflections
2. Inversion of ionograms to determine electron density profile
3. Database of electron density profile parameters and their corresponding reduced ionospheric parameters
4. Simulation of oblique transequatorial ionograms
5. Simulation of ionospheric scintillation and spread-F by temporal variation of electron density profile and creation of plasma bubbles

Future Works

1. Measurement of lithospheric and ionospheric currents using MAGDAS data
2. Maxwell's equation for nonlinear and non-isotropic medium for the magnetohydrodynamics of sun-earth current systems

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