The Sun

Solar physics, Nuclear Physics, Neutrino Physics
Standard Deviation Analysis, Diffusion Entropy Analysis
(space weather phenomena)

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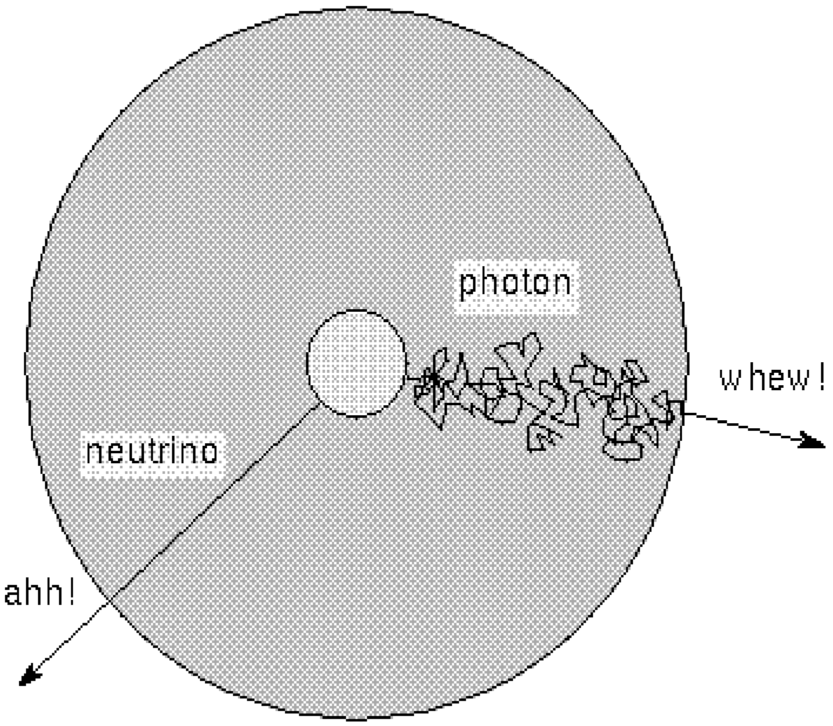
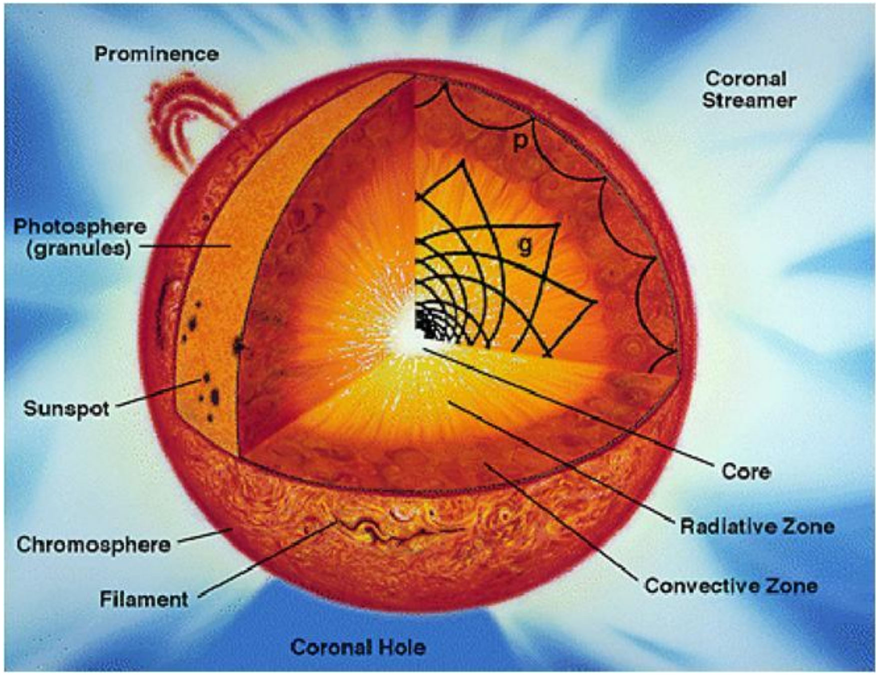
Solar Neutrinos



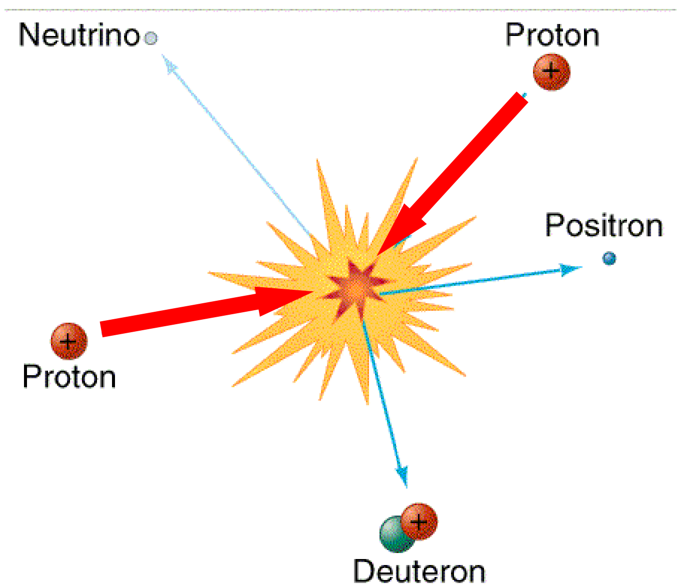
- Gravitationally stabilized solar fusion reactor
- Produced in nuclear fusion reactions
- Exit Sun in about 2 seconds
- Weakly interact with matter (neutrino oscillations)
- Can be detected on Earth
- Flux provides information about solar physics, nuclear physics, and neutrino physics

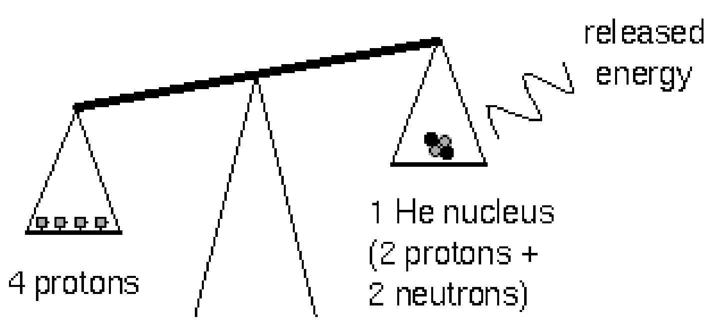
Standard Solar Model

- Begins with homogeneous composition
- Solar core is modeled as an ideal fusion plasma
- Hydrogen burning supplies luminosity and pressure to balance gravity
- Energy is transported by photons
- Chemical composition changes slowly with nuclear reactions
- p-modes and g-modes?
- Magnetic field?



Nuclear Reactions





Some mass is converted into energy ($E=mc^2$)

- Proton- Proton Chain

  - overall:

  $4p \rightarrow 1He + 2e^{+} + 2v_{e} + 25$ MeV

  - reactions produce heavier elements

  - includes sub-chains involving ^{7}Be and ^{8}B

  - accounts for 99.6% of the Sun’s energy

- CNO Cycle

  - accounts for 0.4% of the Sun’s energy

  - dominant in more massive stars

Solar Neutrino Problem

- Observed flux

1/3rd Predicted flux

1. There is some other source of power in the Sun
2. Scientists calculated the reaction rates inaccurately
3. Evidence that neutrinos can change "type" en route to Earth (neutrino oscillations)

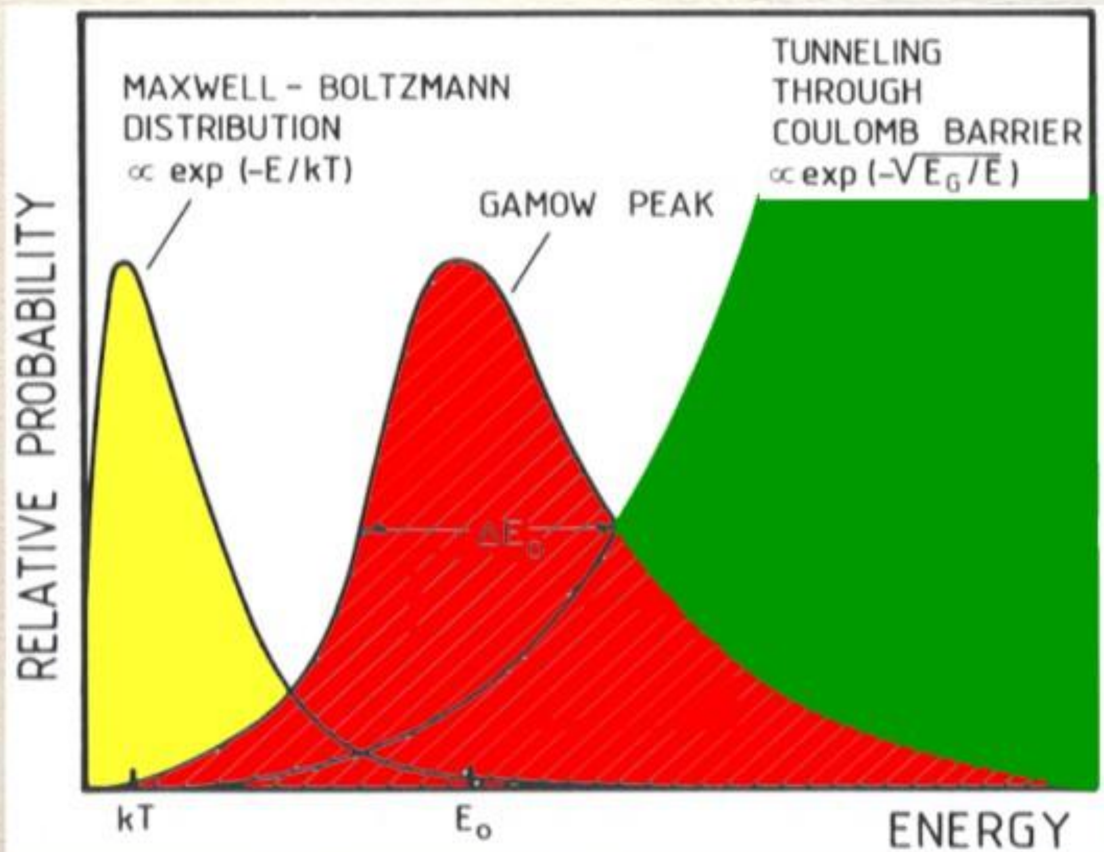
Maxwell-Boltzmann

Maxwell-Boltzmann Distribution

$$\langle \sigma v \rangle = \left(\frac{8}{\mu\pi} \right)^{\frac{1}{2}} \frac{1}{(kT)^{\frac{3}{2}}} \int_0^\infty S(E) \exp\left(\frac{-E}{kT} - \frac{b}{\sqrt{E}} \right) dE$$

$$R(r) = \langle \sigma v \rangle N_a N_x$$

$$\Phi = \frac{1}{4\pi R_{ES}^2} \int_0^{R_{sun}} 4\pi r^2 R(r) dr$$


MAXWELL - BOLTZMANN
DISTRIBUTION
∝ exp (-E/kT)
TUNNELING
THROUGH
COULOMB BARRIER
∝ exp (-√(E_G/E))
GAMOW PEAK
ΔE_0
RELATIVE PROBABILITY
kT
E_0
ENERGY

Tsallis

$$p(E) = \begin{cases} N(\delta, kT)\left[1 - \frac{(2\delta)E}{kT}\right]^{\frac{1}{2\delta}}, E \le \frac{kT}{2\delta} \\ 0, otherwise \end{cases}$$

- Change velocity/energy distribution from Maxwell-Boltzmann to Tsallis
- Vary parameter empirically to match known data

Maxwell-Boltzmann → Tsallis

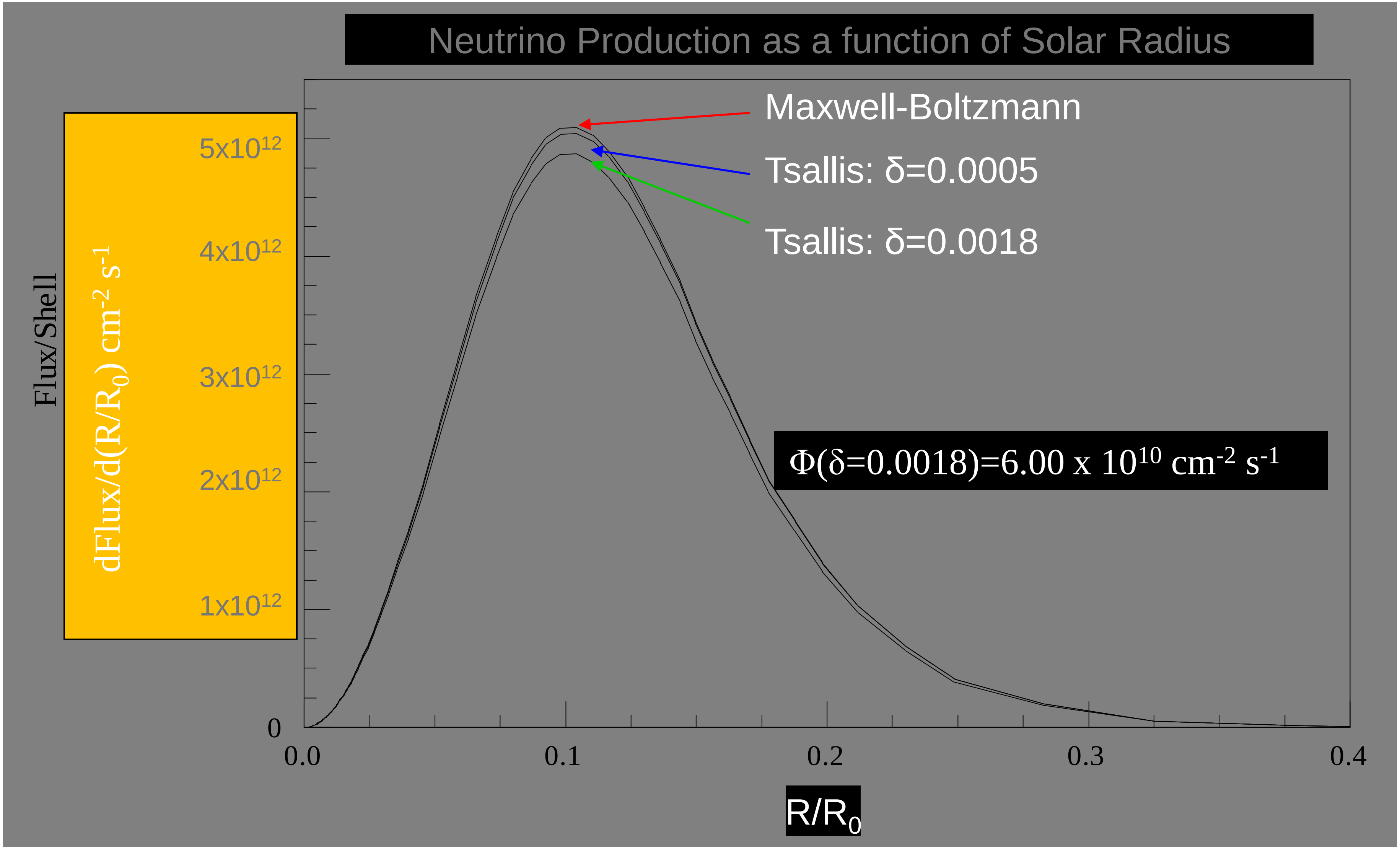
$$\langle \sigma v \rangle = \left(\frac{8}{\mu\pi} \right)^{\frac{1}{2}} \frac{1}{(kT)^{\frac{3}{2}}} \int_0^\infty S(E) \exp\left(\frac{-E}{kT} - \frac{b}{\sqrt{E}} \right) dE$$

$$\langle \sigma v \rangle = \left(\frac{8}{\mu\pi} \right)^{\frac{1}{2}} \frac{1}{(kT)^{\frac{3}{2}}} \int_0^\infty S(E) \exp\left(-\delta \left(\frac{E}{kT} \right)^2 - \frac{E}{kT} - \frac{b}{\sqrt{E}} \right) dE$$

$$R = \langle \sigma v \rangle N_a N_x$$

$$\Phi = \frac{1}{4\pi R_{ES}^2} \int_0^{R_{sun}} 4\pi r^2 R(r) dr$$

PP- Maxwell-Boltzmann vs. Tsallis



Issues to Explore

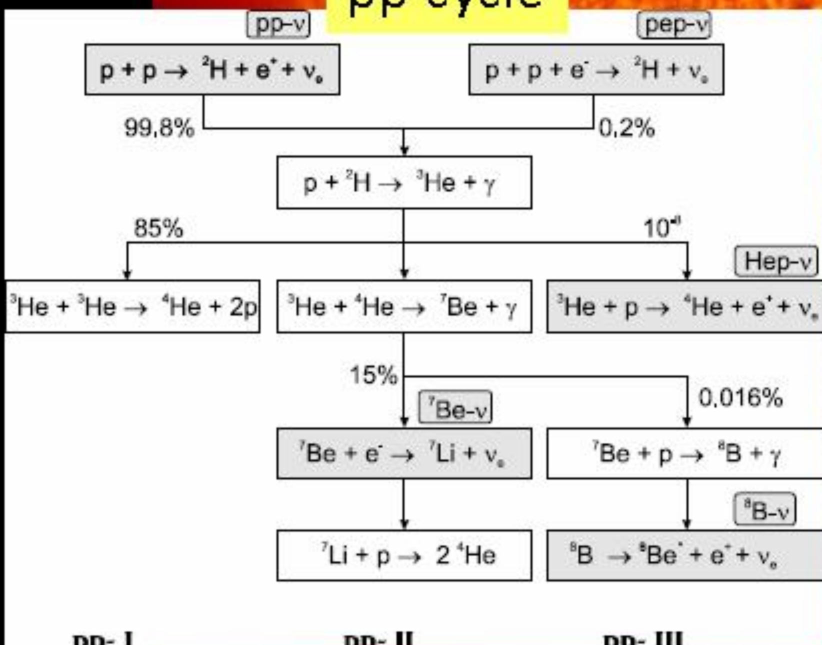
- Find flux for neutrinos from all reactions using Tsallis distribution
- Vary parameter (δ) so that predicted data match observed data
- Implies that the Sun's core is not an ideal plasma
- Magnetic field, g-modes

Solar Neutrinos

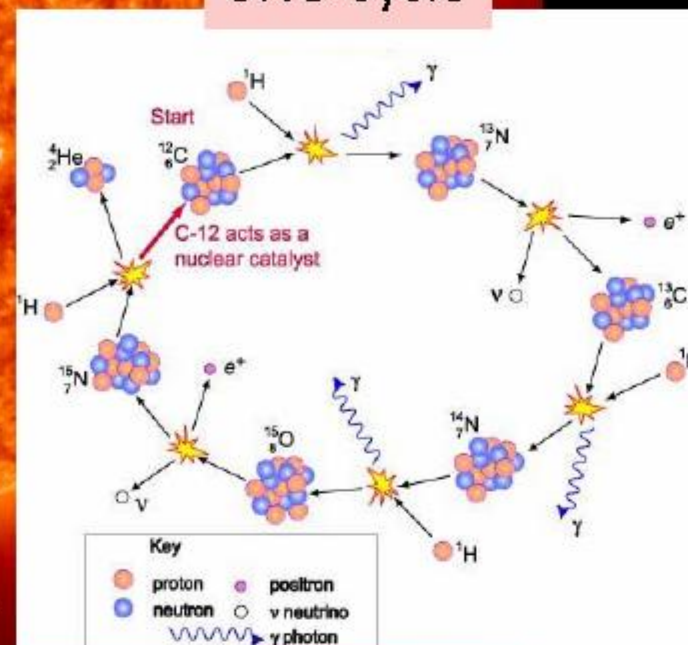
$$4p \rightarrow \mathrm{He}^4 + 2e^+ + 2\nu_e + 26.7\,\mathrm{MeV}$$

$6.5 \cdot 10^{10}\ \nu_e/\mathrm{cm}^2\mathrm{s}$

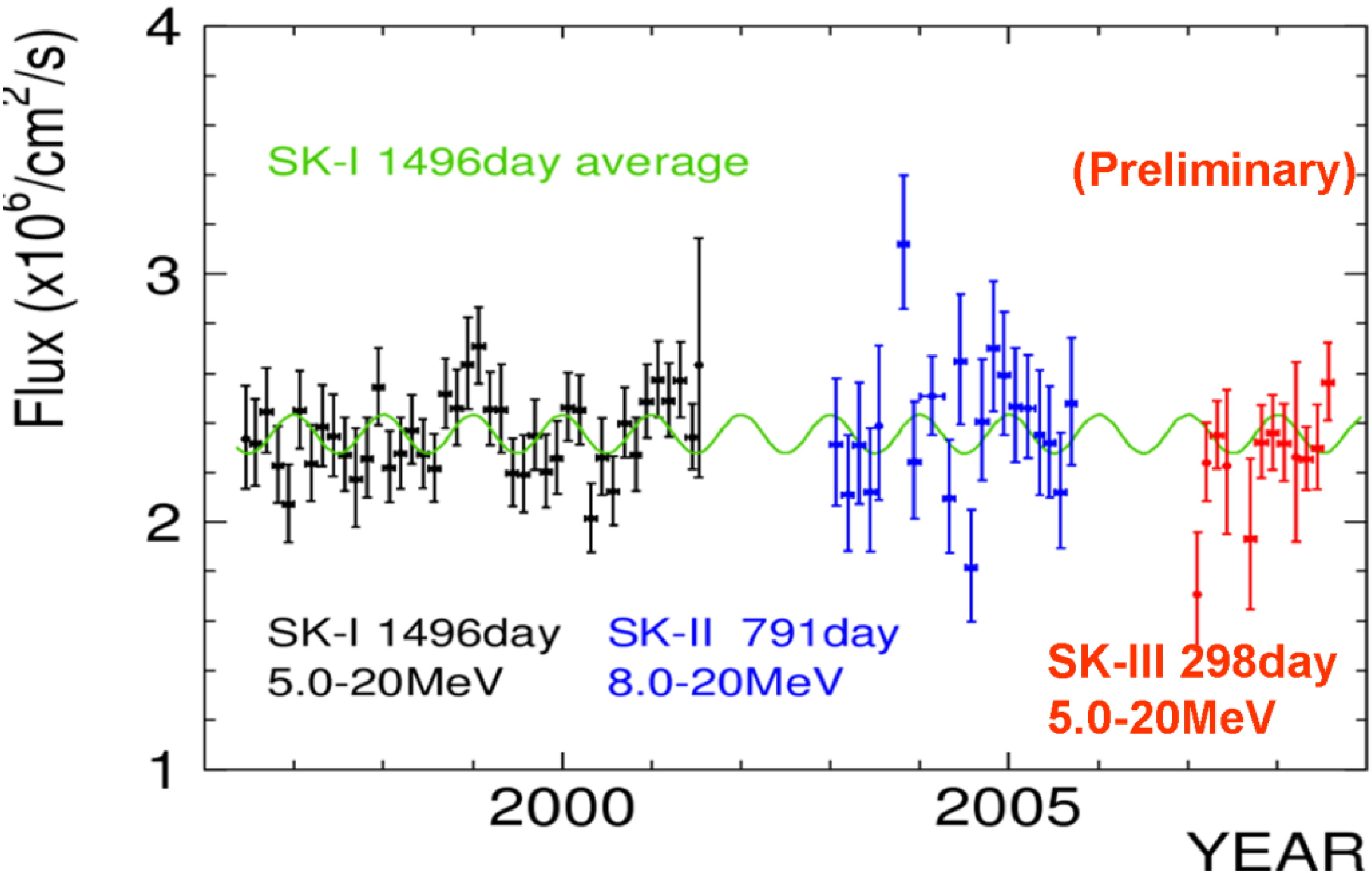
pp cycle



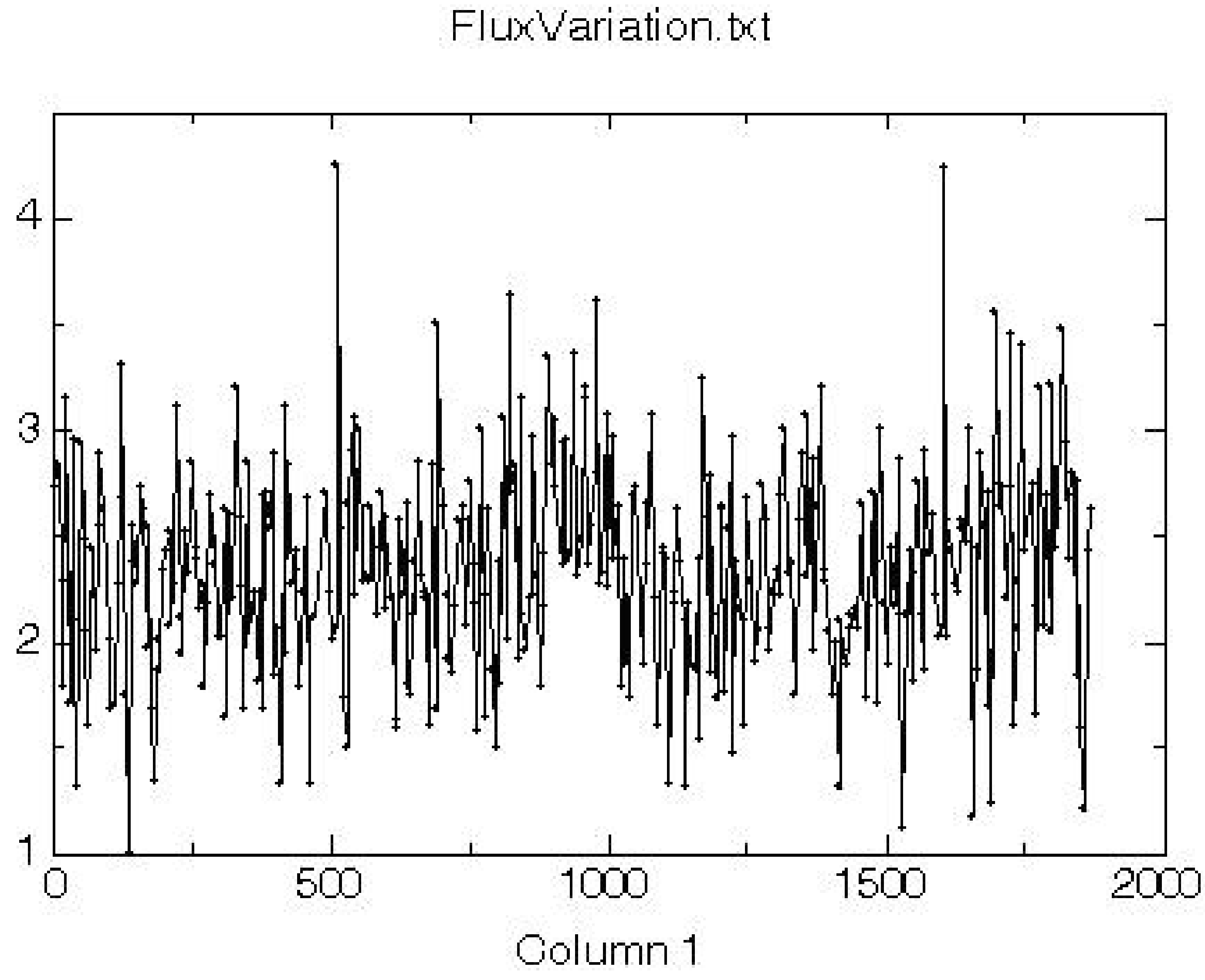
CNO cycle



SuperKamiokande I, II, (III)



Time Series



Mapping Time Series to Diffusion Process

- By summing the terms of a time series we get a trajectory and the trajectory can be used to generate a diffusion process.

Let us consider a time series $\{\xi_i\}$ of N data: $\xi_1, \xi_2, \ldots\ldots, \xi_{N-1}, \xi_N$.
For any given time t, $1 \leq t \leq N$, we can find N-t+1 sub-sequences :

$$\xi_i^{(s)} \equiv \xi_{i+s} \qquad s = 0, 1, 2, \ldots\ldots, N-t$$

✓ For any of these sub-sequence we can build up diffusion trajectory, defined by the position:

$$x^{(s)}(t) = \sum_{i=1}^{t} \xi_i^{(s)} = \sum_{i=1}^{t} \xi_{i+s}$$

Fick’s Laws and Brownian motion

$\vec{J} = -\underline{\underline{D}} \cdot \vec{\nabla} c$	1. Fick‘s Law
$\frac{dc}{dt} = -\underline{\underline{D}} \cdot \vec{\nabla}^2 c$	2. Fick‘s Law

- Einstein unified Fick’s continuum formulation of diffusion with the Stochastic theory and obtained the probability distribution

$$p(x,t) = \frac{1}{\sqrt{4\pi D t}} \exp(-x^2 / 4Dt)$$

Gaussian distribution

Mean square displacement;

$$\left\langle x^2(t) \right\rangle \propto t$$

Anomalous Diffusion

Anomalous diffusion is characterised by

$$\left\langle x^2(t) \right\rangle \propto t^{\gamma} \qquad \text{with } \gamma \neq 1$$

Mandelbrot introduced a distribution to describe anomalous diffusion.

$$p(x,t) = \frac{1}{\sqrt{4\pi D t^{\eta}}} \exp\left(-x^2 \big/ 4Dt^{\eta}\right)$$

Second moment $\langle x^2(t) \rangle = 2Dt^{\eta}$

For $\eta=1$ the normal Brownian motion is recovered. The case $0<\eta<1$ corresponds to Sub diffusion and $\eta>1$ corresponds to Super diffusion.

Scaling Analysis of Time Series

- A diffusion process with scaling can be described by the probability density function (*pdf*)

$$p(x,t) = \frac{1}{t^{\delta}} F\left(\frac{x}{t^{\delta}}\right)$$

δ is called the *pdf* scaling exponent.

✓ The *Variance Scaling exponent* H of a diffusion process is defined by:

$$\Sigma^2(t) \sim t^{2H}$$

If $<x(t)>=0$, the variance, $\Sigma^2(t)=<x^2(t)>-<x(t)>^2$ coincides with the mean squared displacement. Then

$$\sum{}^2(t) = < x^2(t) > = \int_{-\infty}^{\infty} x^2 p(x,t)\,dx \sim t^{2H}$$

Scaling Analysis of Time Series

Now $p(x,t) = \frac{1}{t^{\delta}} F(\frac{x}{t^{\delta}})$

$$< x^2(t) >= \int_{-\infty}^{\infty} x^2 \frac{1}{t^{\delta}} F(\frac{x}{t^{\delta}}) dy$$

$$= t^{2\delta} \int_{-\infty}^{\infty} y^2 F(y) dy$$

Where $y=x/t^{\delta}$

If $\int_{-\infty}^{\infty} y^2 F(y) dy =$ Constant $<\infty$, $<x^2(t)>\sim t^{2\delta}$

➢ Thus the *pdf scaling exponent* δ and the *variance scaling exponent* H coincide in all cases with $\int y^2 F(y)dy=\text{constant}<\infty$.

✓ This holds true, for example, in the normal Brownian motion.

Diffusion Entropy Analysis

- "Diffusion Entropy Analysis (DEA)" finds out the pdf scaling exponent δ.

- DEA is based on the evaluation of the Shannon entropy of the diffusion process.

Shannon entropy:

$$S(t) = -\int_{-\infty}^{\infty} p(x,t)\ln[p(x,t)]dx$$

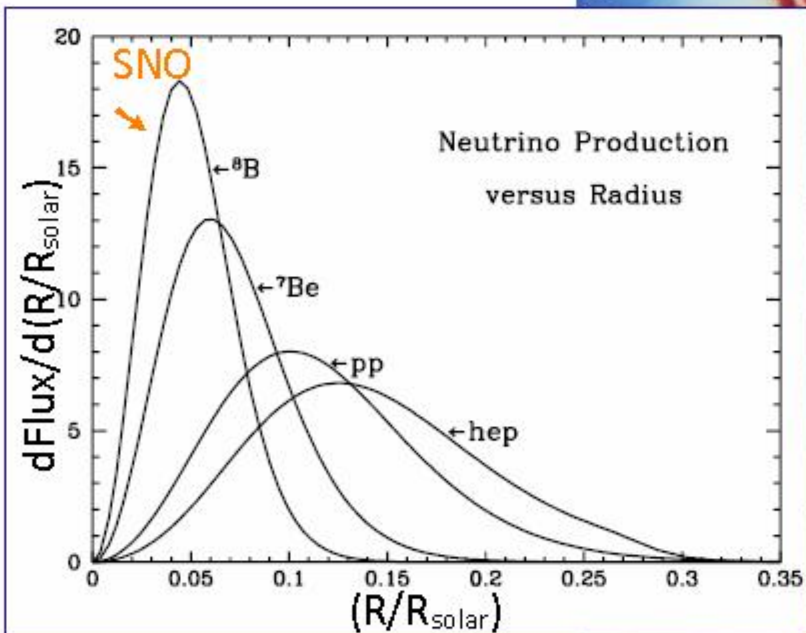
Using the scaling pdf we get, $S(t)=A+\delta\ln(t)$

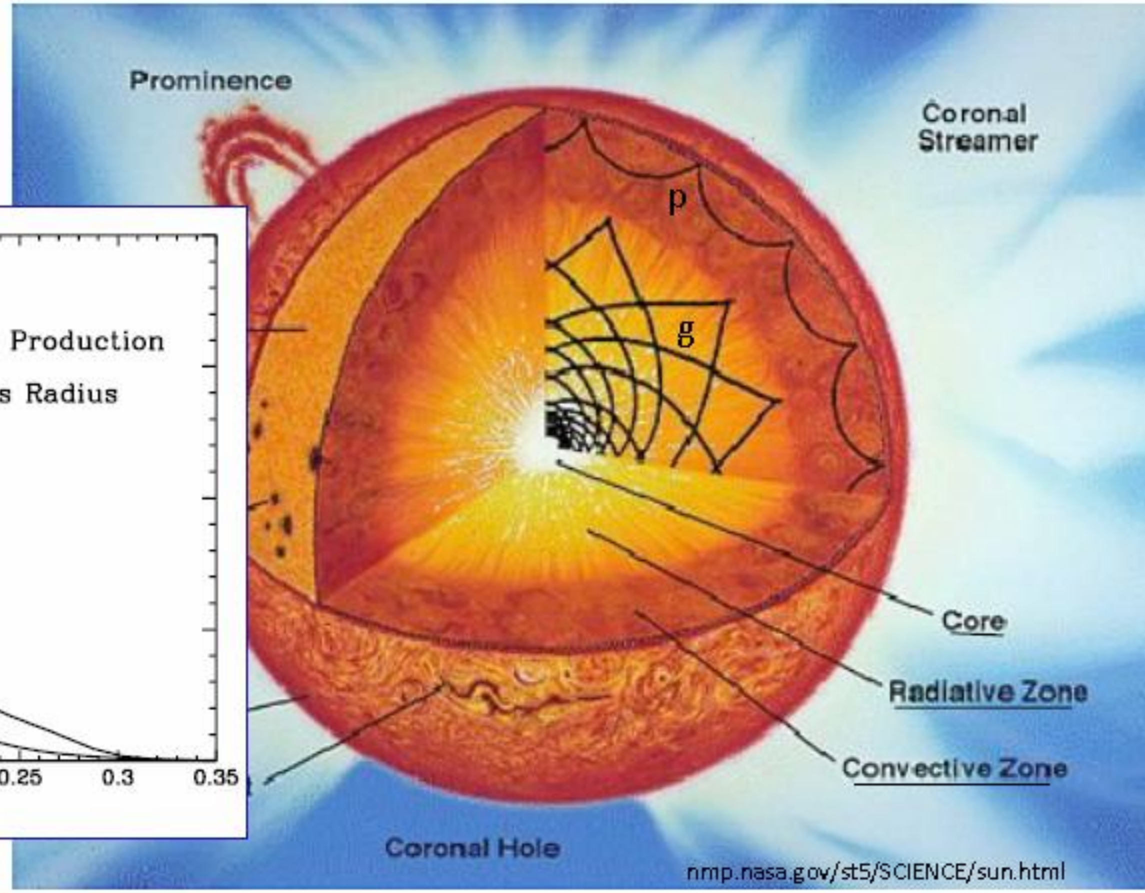
✓ The slope of the log-linear plot of *S(t)* against *t* gives the *pdf* scaling exponent δ.

Relation between two scaling exponents

- $H = \delta = 0.5$: Normal diffusion
- $H = \delta \neq 0.5$: Fractional Brownian motion
- $H \neq \delta$: Levy Flight
- $\delta = 1/(3-2H)$: Levy Walk

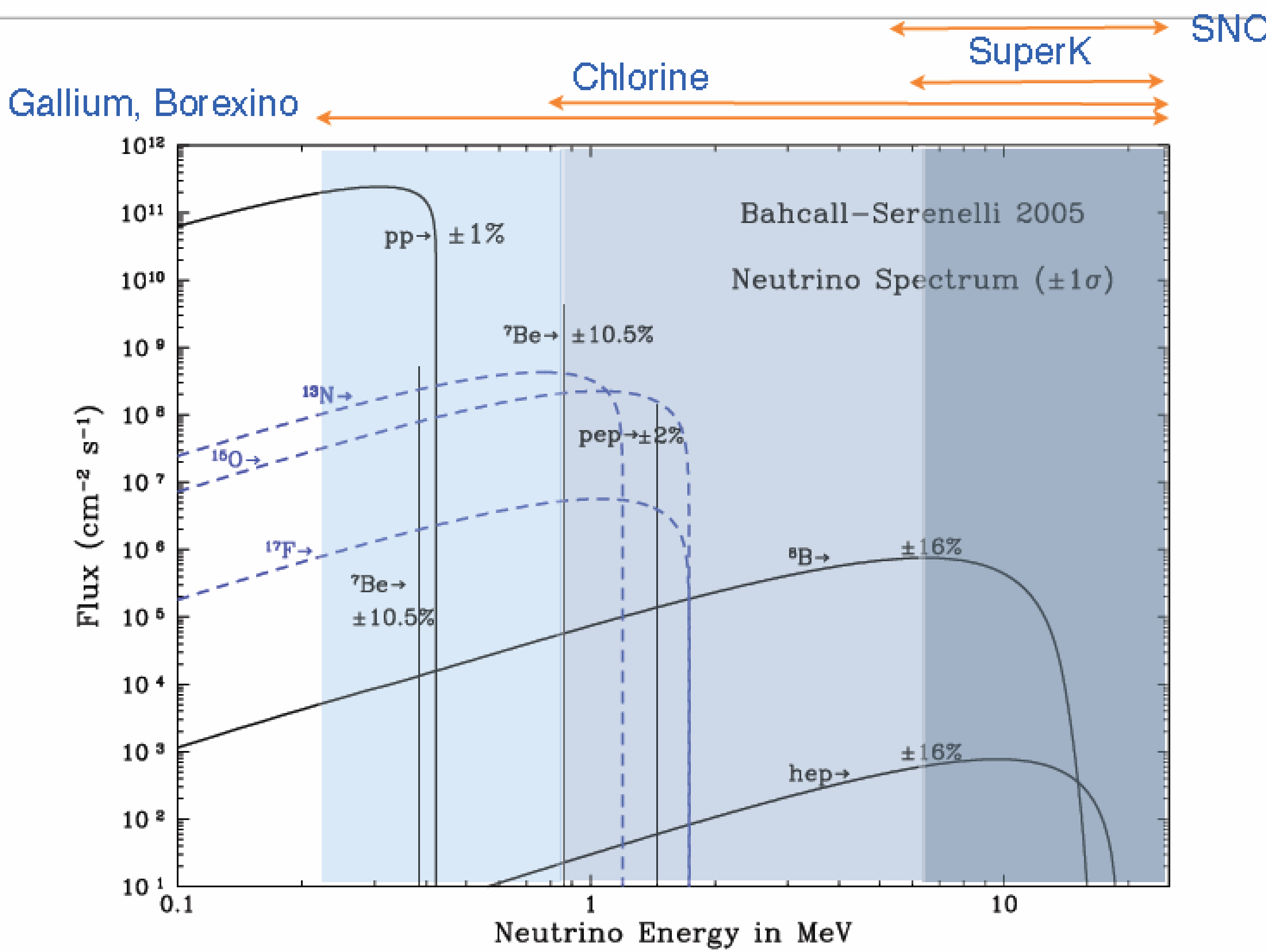
Standard Solar Model: solar oscillations



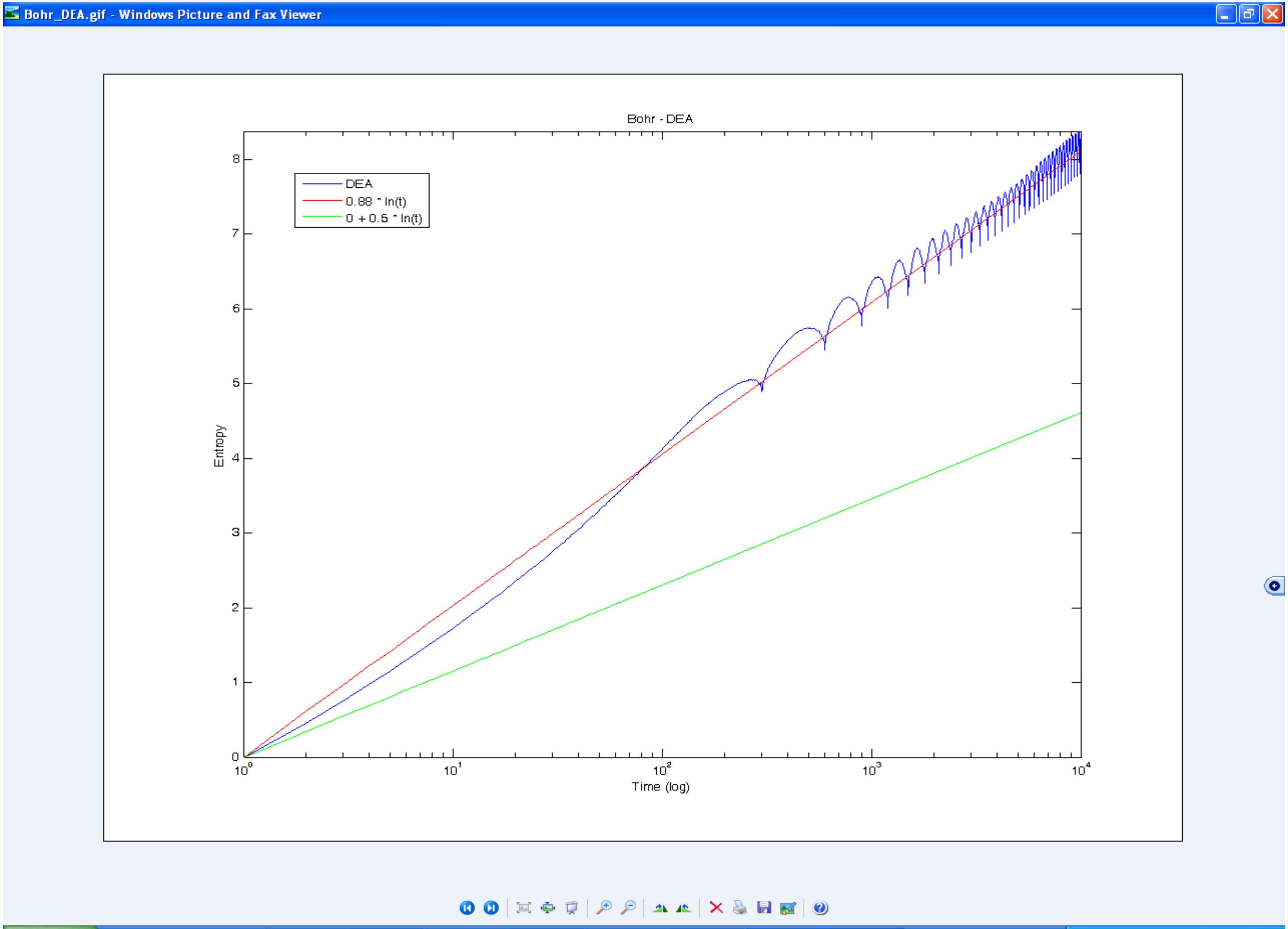


Neutrino production: core
p-mode (acoustic) oscillations: largely convective zone
g-mode (gravity) waves: core and radiative zone

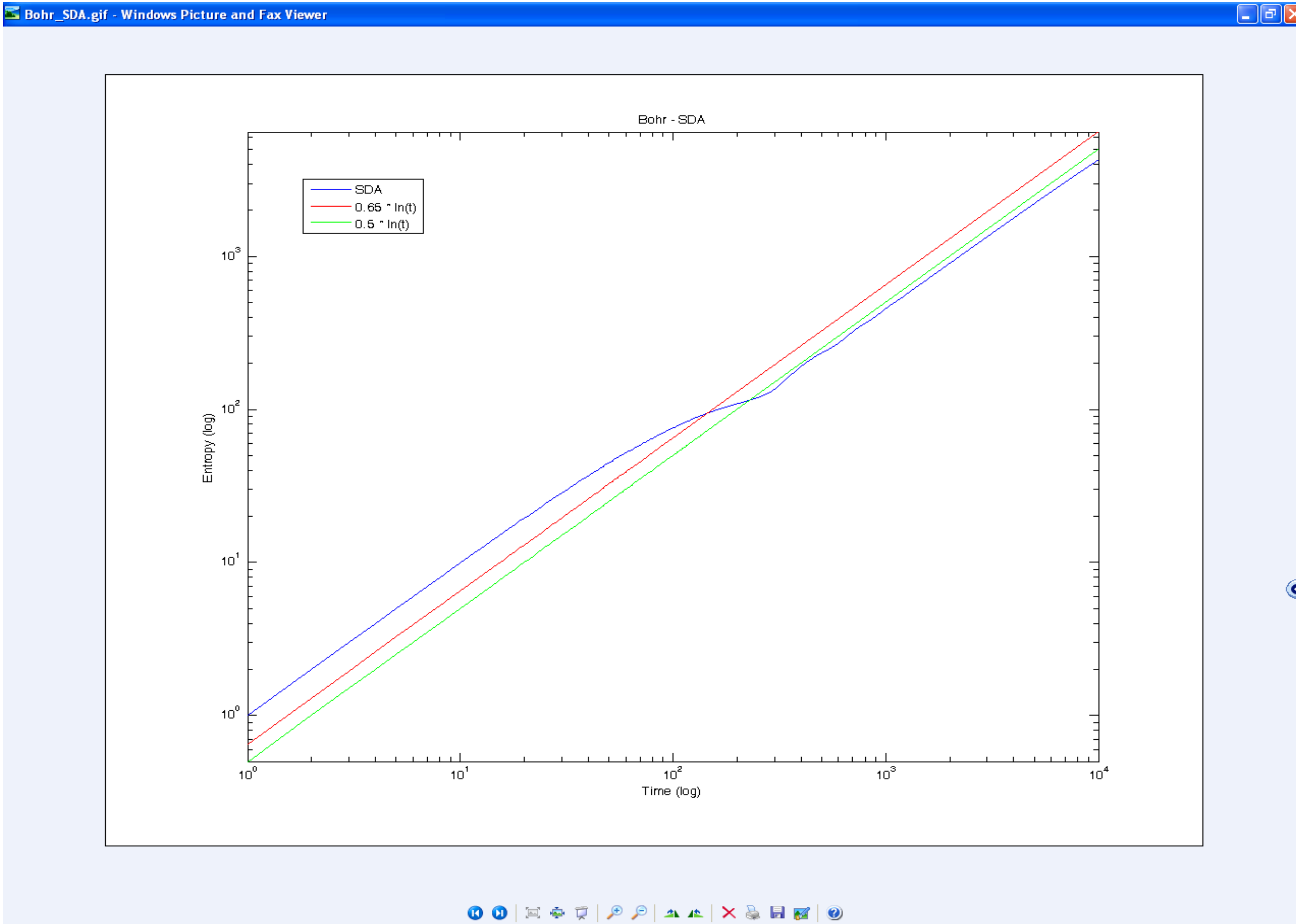
The solar neutrino spectrum

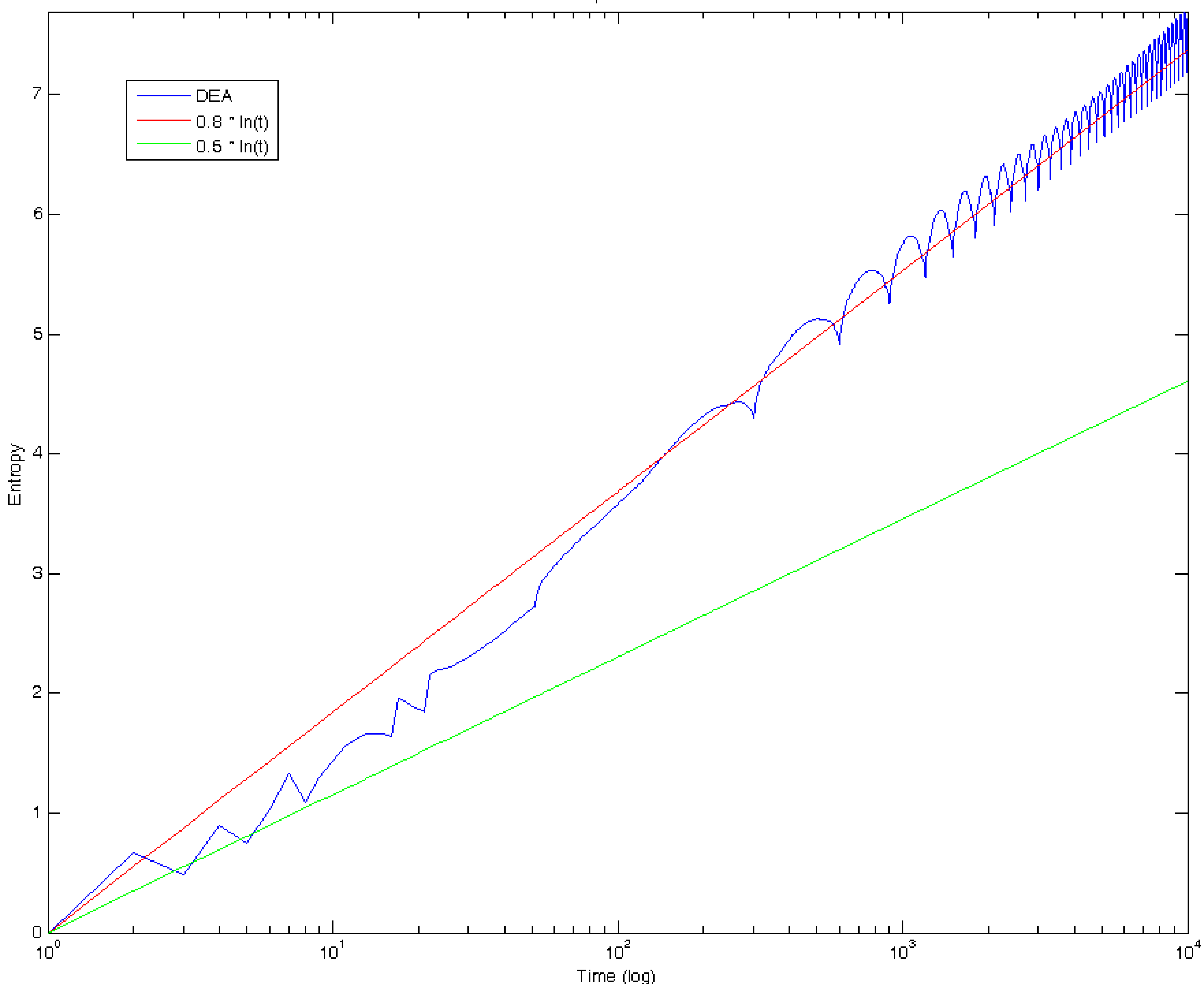


DEA: delta=0.88

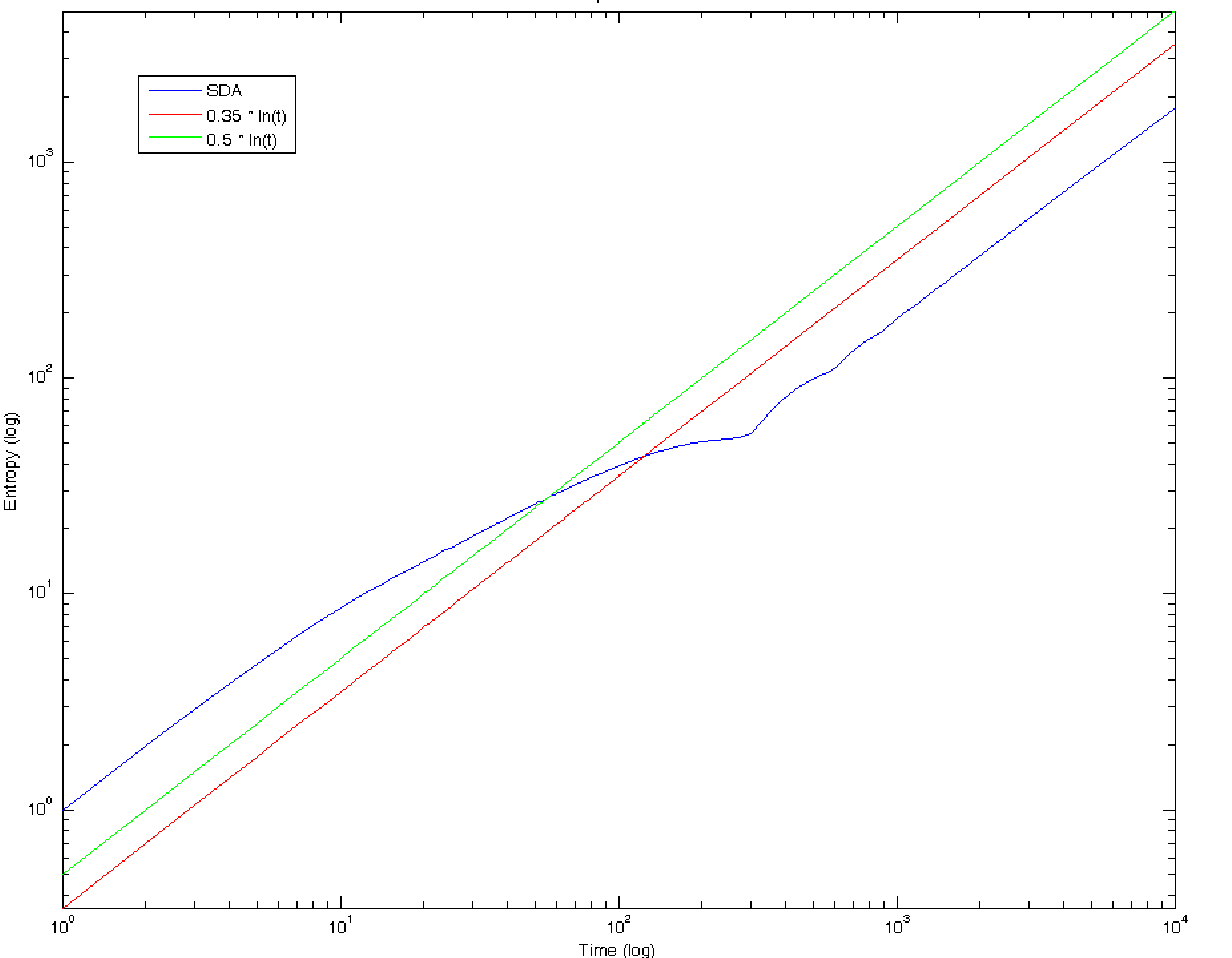


SDA: H=0.66


Bohr_SDA.gif - Windows Picture and Fax Viewer
Bohr - SDA
SDA
0.65 * ln(t)
0.5 * ln(t)
Entropy (log)
Time (log)
10^0
10^1
10^2
10^3
10^4



DEA
0.8 * ln(t)
0.5 * ln(t)
Entropy
Time (log)
0
1
2
3
4
5
6
7
10^0
10^1
10^2
10^3
10^4



SDA
0.35 * ln(t)
0.5 * ln(t)
Entropy (log)
Time (log)
10^0
10^1
10^2
10^3
10^4

Shannon Entropy rather than Mean Squared Displacement!
(DEA-pdf versus SDA-variance?)
SK solar neutrino signal is not GAUSSian
(TSALLIS, LÉVY, …?)
SK solar neutrino signal shows periodic modulation
(g-modes, magnetic field, …?)

Thank you for your attention!

- United Nations Office for Outer Space Affairs
- Centre for Mathematical Sciences India