Three-fluid dynamics and generalized Ohm's law for understanding Magnetosphere-Ionosphere coupling

Akimasa Yoshikawa

International Center for Space Weather Science and Education (ICSWSE)
Department of Earth and Planetary Sciences
Kyushu University, Japan

Conventional approach

Ionosphere model by Ohmic current region

$$\mathbf{j}_\perp = \sigma_P\left(\mathbf{E}+\mathbf{u}_n\times\mathbf{B}_0\right)+\sigma_H\hat{e}_B\times\left(\mathbf{E}+\mathbf{u}_n\times\mathbf{B}_0\right)$$

E-field in the neutral frame $\mathbf{E}+\mathbf{u}_n\times\mathbf{B}_0$

σ_P Pedersen conductivity

σ_H Hall conductivity

- derived from charged particle mobility, under presence of electric field
- connect it to field-aligned current through div(j)=0
- FAC is carried by shear Alfven wave
- coupling (**B**,**V**) description of magnetosphere and (**E**,**J**) description of is required

However, in this formulation, we cannot well understand
how magnetosphere and ionosphere are seamlessly coupled!!
Especially, in the context of global (**B**,**V**) description

Outline

(1) Fluid description (**B**,**V**) for partially ionized system

- Plasma Equation of motions
- Generalized Ohm's law

(2) M-I coupling via shear Alfven wave

- magnetospheric dynamo
- wave generation and propagation and their relation to velocity shear
- wave propagation inside the ionosphere
- atmospheric dynamo
- 3D current closure inside ionosphere

Fundamental evolution equations of physical quantities Q_k

$$\frac{\partial}{\partial t} Q_k = F_k(Q_1, Q_2, Q_3, \ldots)$$

$$Q_k = \mathbf{B}, \mathbf{E}, \mathbf{J}, \mathbf{V}, \mathbf{V}_n$$

What's drive What in the weakly ionized system?

evolution of B-field $$\frac{\partial}{\partial t}\mathbf{B} = -(\nabla \times \mathbf{E})$$

evolution of E-field $$\frac{\partial}{\partial t}\mathbf{E} = \frac{(\nabla \times \mathbf{B}) - \mu_0 \mathbf{j}}{\varepsilon_0 \mu_0}$$

evolution of current density $$\frac{\partial}{\partial t}\mathbf{j} = ?$$

evolution of plasma velocity $$\frac{\partial}{\partial t}\mathbf{V} = ?$$

evolution of neutral velocity $$\frac{\partial}{\partial t}\mathbf{V}_n = ?$$

momentum equations in the collisional system

for ion fluid

$$m_i n_i \frac{\partial}{\partial t}\mathbf{v}_i = e n_i\left(\mathbf{E}+\mathbf{v}_i\times\mathbf{B}\right)-\nabla\cdot p_i - m_i \nu_{in} n_i\left(\mathbf{v}_i-\mathbf{v}_n\right)-m_e\nu_{ei}n_i\left(\mathbf{v}_i-\mathbf{v}_e\right) \qquad (1)$$

Lorentz force

kinetic tensor

momentum exchange between plasma and neutral

momentum exchange between electron and ion

for electron fluid

$$m_e n_e \frac{\partial}{\partial t}\mathbf{v}_e = -e n_e\left(\mathbf{E}+\mathbf{v}_e\times\mathbf{B}\right)-\nabla\cdot p_e - m_e \nu_{en} n_e\left(\mathbf{v}_e-\mathbf{v}_n\right)+m_e\nu_{ei}n_i\left(\mathbf{v}_i-\mathbf{v}_e\right) \qquad (2)$$

for neutral fluid

momentum exchange Between ion and neutral

momentum exchange Between electron and neutral

$$\rho_n\left[\frac{\partial \mathbf{v}_n}{\partial t}+(\mathbf{v}_n.\nabla)\mathbf{v}_n\right]=-\nabla p_n-\rho_n\nu_{ni}(\mathbf{v}_n-\mathbf{V}_i)-\rho_n\nu_{ne}(\mathbf{v}_n-\mathbf{V}_e)+\mathbf{F}_n \qquad (3)$$

One fluid–description for global phenomena (Hall MHD approximation)

plasma density

$$n = n_i = n_e$$

pressure gradient

$$P = P_i + P_e$$

current density

$$\mathbf{j} = en\left(\mathbf{v}_i - \mathbf{v}_e\right)$$

(+) << (−)

barycentric velocity

$$\mathbf{V} = \frac{m_i \mathbf{v}_i + m_e \mathbf{v}_e}{m_i + m_e}$$

(+) >> (−)

mass density

$$\rho = \left(m_i + m_e\right)n$$

(+) >> (−)

$$m_i >> m_e$$

ion velocity

$$\mathbf{v}_i = \mathbf{V} + \left(\frac{m_e}{\rho e}\right)\mathbf{j}$$

electron velocity

$$\mathbf{v}_e = \mathbf{V} - \left(\frac{m_i}{\rho e}\right)\mathbf{j}$$

One-fluid description of ion and electron (motion and current)

○ dominant part

↕ cancellation part

equation of motion

for ion

$$m_i n \frac{d}{dt}\left[\mathbf{V}+\left(\frac{m_e}{\rho e}\right)\mathbf{j}\right]=en(\mathbf{E}+\mathbf{V}\times\mathrm{B})+\left(\frac{m_e}{m_i+m_e}\right)\mathbf{j}\times\mathbf{B}-m_i n\nu_{in}\left[\mathbf{V}-\mathbf{v}_n-\left(\frac{m_e}{\rho e}\right)\mathbf{j}\right]-\left(\frac{m_i\nu_{ie}}{e}\right)\mathbf{j}-\nabla p_i$$

for electron

$$m_e n \frac{d}{dt}\left[\mathbf{V}-\left(\frac{m_i}{\rho e}\right)\mathbf{j}\right]=-en(\mathbf{E}+\mathbf{V}\times\mathrm{B})+\left(\frac{m_i}{m_i+m_e}\right)\mathbf{j}\times\mathbf{B}-m_e n\nu_{en}\left[\mathbf{V}-\mathbf{v}_n+\left(\frac{m_i}{\rho e}\right)\mathbf{j}\right]-\left(\frac{m_e\nu_{ei}}{e}\right)\mathbf{j}-\nabla p_e$$

inertial part — Lorentz force — Ampere force — momentum exchange to neutrals — momentum exchange between ion and electrons

current density

ionic current

$$en\frac{d}{dt}\left[\mathbf{V}+\left(\frac{m_e}{\rho e}\right)\mathbf{j}\right]=\left(\frac{e^2 n}{m_i}\right)(\mathbf{E}+\mathbf{V}\times\mathrm{B})+e\left(\frac{m_e}{m_i}\right)\frac{\mathbf{j}\times\mathbf{B}}{m_i+m_e}-en\nu_{in}\left[\mathbf{V}-\mathbf{v}_n-\left(\frac{m_e}{\rho e}\right)\mathbf{j}\right]-\nu_{ie}\mathbf{j}-\frac{e\nabla p_i}{m_i}$$

electron current

$$-en\frac{d}{dt}\left[\mathbf{V}-\left(\frac{m_i}{\rho e}\right)\mathbf{j}\right]=\left(\frac{e^2 n}{m_e}\right)(\mathbf{E}+\mathbf{V}\times\mathrm{B})-e\left(\frac{m_i}{m_e}\right)\frac{\mathbf{j}\times\mathbf{B}}{m_i+m_e}+en\nu_{en}\left[\mathbf{V}-\mathbf{v}_n+\left(\frac{m_i}{\rho e}\right)\mathbf{j}\right]+\nu_{ei}\mathbf{j}+\frac{e\nabla p_e}{m_e}$$

ion : main portion of plasma dynamics

electron : main portion of current carier

Equation for motion of plasma

inertial — pressure gradient — friction force along j

$$\rho \frac{\partial}{\partial t}\mathbf{V} = \mathbf{j}\times\mathbf{B} - \nabla P - n\left(m_i \nu_{in} + m_e \nu_{en}\right)\left(\mathbf{V} - \mathbf{V}_n\right) - m_e\left(\nu_{in} - \nu_{en}\right)\frac{\mathbf{j}}{e}$$

Ample force — momentum exchange between neutral and charged particles

Generalized Ohm's law

electromotive e-field

$$\frac{\partial}{\partial t}\mathbf{j} = \varepsilon_0 \omega_{pe}^2\left(\mathbf{E} + \overrightarrow{emf} - \overleftrightarrow{R}\cdot\mathbf{j}\right)$$

reactive-field

convection electric field — am-bipolar field

$$\overrightarrow{emf} = \mathbf{V}\times\mathbf{B} + \frac{B\left(\nu_{in} - \nu_{en}\right)}{\Omega_e}\left(\mathbf{V} - \mathbf{v}_n\right) - \frac{\nabla p_e}{ne}$$

electric field induced by momentum exchange effect

$$\overleftrightarrow{R}\cdot\mathbf{j} = \left(\frac{B}{ne}\right)\left[\left(\frac{\nu_{en} + \nu_{ei}}{\Omega_e}\right)\mathbf{j} + \left(\mathbf{j}\times\hat{\mathbf{b}}\right)\right]$$

resistive e-field — Hall e-field

Distribution of collision frequency

controls the momentum exchange between neutral and charged particles

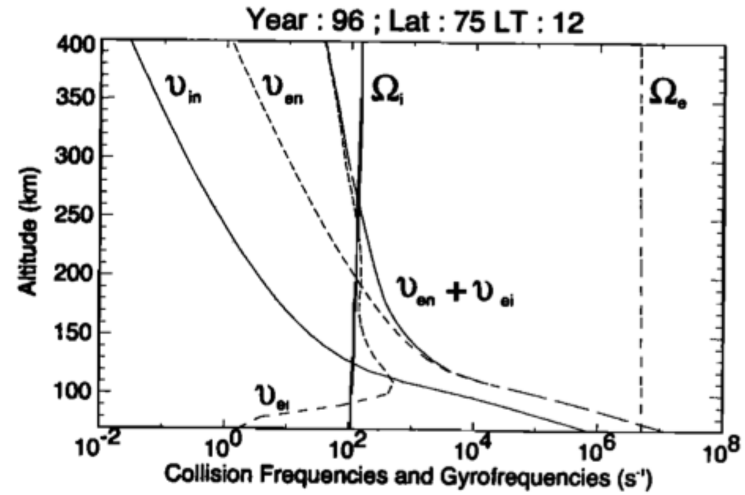
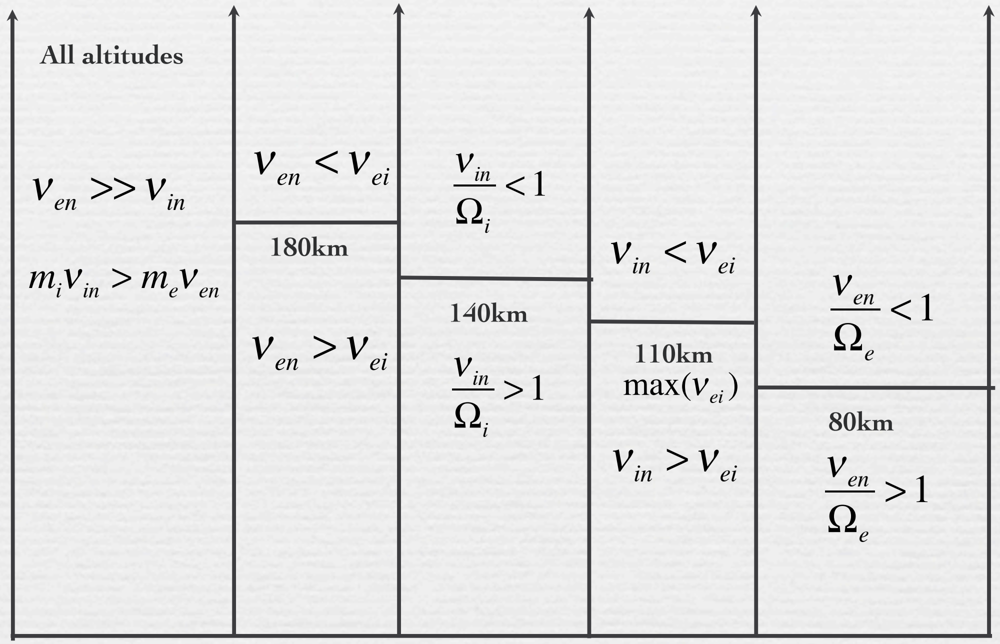


Figure 1. Collision frequencies and gyrofrequencies as functions of altitude at local noon at 75° latitude. They are determined based on the observations/laboratory experiments and the formula of *Kelley* [1989]. The anisotropy in the collision frequencies become unimportant above 80 km. More detailed discussion on the relationships among these quantities and anisotropies is given by *Richmond* [1995].

From Song, et al., [2001]



Equation for motion of plasma (dominant)

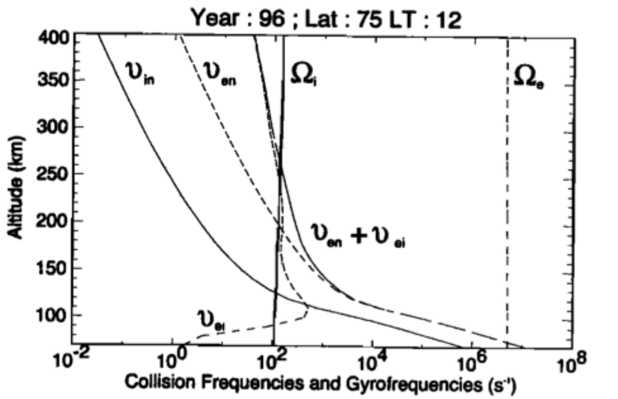


Figure 1. Collision frequencies and gyrofrequencies as functions of altitude at local noon at 75° latitude. They are determined based on the observations/laboratory experiments and the formula of *Kelley* [1989]. The anisotropy in the collision frequencies become unimportant above 80 km. More detailed discussion on the relationships among these quantities and anisotropies is given by *Richmond* [1995].

$$m_i \nu_{in} >> m_e \nu_{en}$$

$$\nu_{in} << \nu_{en}$$

$$\frac{\partial}{\partial t}\mathbf{V} = -\nu_{in}(\mathbf{V} - \mathbf{V}_n) + \frac{\mathbf{j}\times\mathbf{B}}{\rho} + \frac{B}{\rho}\left(\frac{\nu_{en}}{\Omega_e}\right)\mathbf{j} - \frac{\nabla P}{\rho}$$

- inertial: important ω>~νin
- momentum exchange between ion and neutrals: dominant in the ionosphere
- Ampere force: everywhere important
- friction force along j: important below 80km
- pressure gradient force: Important near plasma sheet

Generalized Ohm's law (dominant)

Current evolution equation

$$\frac{\partial}{\partial t}\mathbf{j} = \varepsilon_0 \omega_{pe}^2 \left(\mathbf{E} + \overrightarrow{emf} - \overleftrightarrow{R}\cdot\mathbf{j}\right)$$

e-field

Electromotive e.m.f. e-field

$$\overrightarrow{emf} = \mathbf{V}\times\mathbf{B} - B\left(\frac{\nu_{en}}{\Omega_e}\right)\left(\mathbf{V} - \mathbf{v}_n\right)$$

convection e- field

e-field induced by momentum exchange effect

reactive-field

$$\overleftrightarrow{R}\cdot\mathbf{j} = \left(\frac{B}{ne}\right)\left[\left(\frac{\nu_{en}}{\Omega_e}\right)\mathbf{j}_\perp + \left(\mathbf{j}\times\hat{\mathbf{b}}\right)\right]$$

resistive e-field Hall e-field

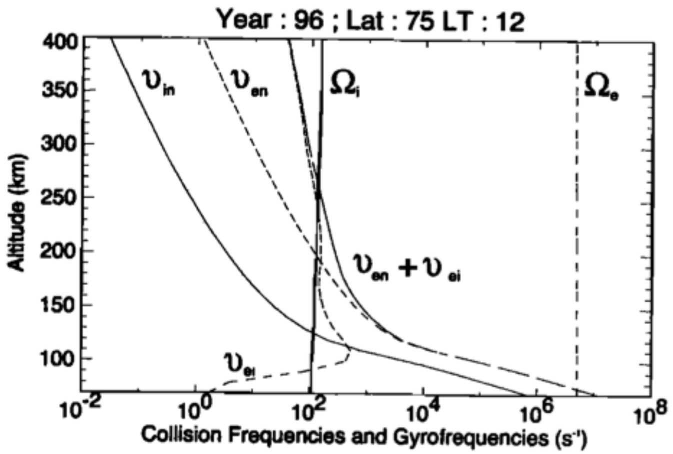


Figure 1. Collision frequencies and gyrofrequencies as functions of altitude at local noon at 75° latitude. They are determined based on the observations/laboratory experiments and the formula of *Kelley* [1989]. The anisotropy in the collision frequencies become unimportant above 80 km. More detailed discussion on the relationships among these quantities and anisotropies is given by *Richmond* [1995].

Electromagnetic wave Radiation

evolution of B-field

$$\frac{\partial}{\partial t}\mathbf{B} = -\left(\nabla \times \mathbf{E}\right)$$

evolution of E-field

$$\frac{\partial}{\partial t}\mathbf{E} = \frac{\left(\nabla \times \mathbf{B}\right) - \mu_0 \mathbf{j}}{\varepsilon_0 \mu_0}$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} + \nabla \times \left(\nabla \times \mathbf{E}\right) = -\mu_0 \frac{\partial}{\partial t}\mathbf{j}$$

evolution eq of **J**?

no!! electromagnetic field radiations eq by changing of current
(like a dipole antenna)

we need to know how **J** is produced by electro-dynamics (not -magnetics)

$$\frac{\partial}{\partial t}\mathbf{j} = \varepsilon_0 \omega_{pe}^2 \left(\mathbf{E} - \overrightarrow{emf} - \overleftrightarrow{R}\mathbf{j}\right)$$

$$\overleftrightarrow{R}\mathbf{j} = \left[\frac{B}{ne}\right]\left[\left(\frac{\nu_{en} + \nu_{ei}}{\Omega_e}\right)\mathbf{j} + \mathbf{j} \times \hat{\mathbf{b}}\right]$$

After radiation of plasma waves

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E}+\nabla\times(\nabla\times\mathbf{E})=-\mu_0\frac{\partial}{\partial t}\mathbf{j}$$

$$\frac{\partial}{\partial t}\mathbf{j}=\varepsilon_0\omega_{pe}^2\left\{\mathbf{E}+\overrightarrow{emf}-\left[\frac{B}{ne}\right]\left[\left(\frac{\nu_{en}+\nu_{ei}}{\Omega_e}\right)\mathbf{j}+\mathbf{j}\times\hat{\mathbf{b}}\right]\right\}$$

$$\mathbf{j}=\frac{(\nabla\times\mathbf{B})}{\mu_0}-\varepsilon_0\frac{\partial}{\partial t}\mathbf{E}$$

$\sim\frac{\lambda_e^2}{L^2}\mathbf{E}$ $\qquad$ $\sim(\omega_{pe}T)^{-2}\mathbf{E}$ $\qquad$ $\sim(\omega_{pe}T)^{-1}\left(\frac{\Omega_e}{\omega_{pe}}\right)\mathbf{E}$ $\qquad$ $(\omega_{pe}T)^{-1}\left(\frac{\nu_{en}-\nu_{ei}}{\omega_{pe}}\right)\mathbf{E}$

$$-\lambda_e^2\nabla\times(\nabla\times\mathbf{E})-\omega_{pe}^{-2}\frac{\partial^2}{\partial t^2}\mathbf{E}+\left(\frac{\Omega_e}{\omega_{pe}^2}\right)\frac{\partial}{\partial t}(\mathbf{E}\times\hat{e}_B)+\left(\frac{\nu_{en}-\nu_{ei}}{\omega_{pe}^2}\right)\frac{\partial}{\partial t}\mathbf{E}=\mathbf{E}-\overrightarrow{emf}-\mu_0^{-1}\overleftrightarrow{R}\cdot(\nabla\times\mathbf{B})$$

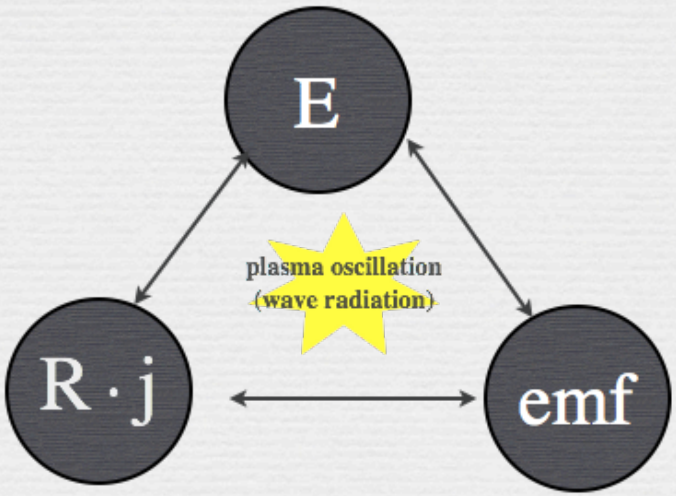
$L>>\lambda_e$

much larger than electron inertial length

$T>>\omega_{pe}^{-1}$

much slower than plasma oscillation

$$\mathbf{E}=\overrightarrow{emf}+\mu_0^{-1}\overleftrightarrow{R}\cdot(\nabla\times\mathbf{B})$$



after electromagnetic wave radiation

balance equation among E, emf and reactive fields : (Steady Ohm's law) in (B,V) scheme is established

Reduced (fundamental) eqs from magnetosphere to ionospheric E-region

$$\rho \frac{d}{dt}\mathbf{V} = \mathbf{j}\times\mathbf{B} - \nabla p - \rho \nu_{in}(\mathbf{V} - \mathbf{u}_n)$$

magnetosphere $\nu_{in}^{-1} >> \Omega_i^{-1}$

inside E-region

Every where important for plasma

$$\mathbf{E} \cong -\mathbf{v}\times\mathbf{B} - \frac{\mathbf{j}\times\mathbf{B}}{en}$$

plasma frozen-in condition

Same as electron frozen-in condition
(ion demagnetization)

$$\mathbf{E} + \mathbf{v}_e \times \mathbf{B} = 0$$

$$\frac{d}{dt}\left(\frac{\rho \mathbf{v}^2}{2}\right) = (\mathbf{j}\times\mathbf{B})\cdot\mathbf{v} - \nabla p\cdot\mathbf{v} - \rho\nu_{in}(\mathbf{v} - \mathbf{u}_n)\cdot\mathbf{v}$$

$$(\mathbf{j}\times\mathbf{B})\cdot\mathbf{v} = \mathbf{j}\cdot\mathbf{E} = \nabla\cdot\mathbf{S}$$

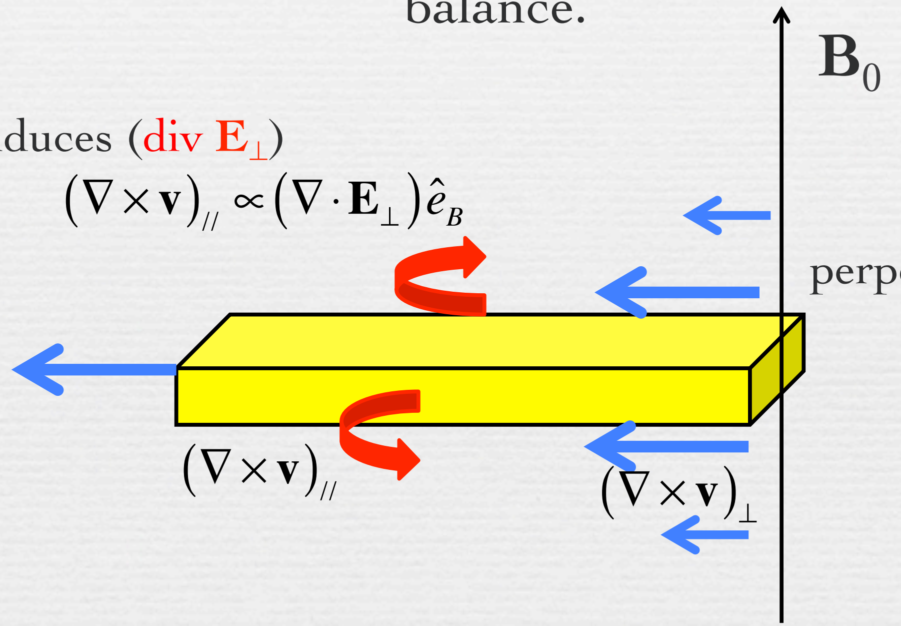
Magnetospheric Generator

"Generator" region can be defined as a region enable to production of **velocity shear (vorticity)** both parallel and perpendicular direction to $\mathbf{B}_0$ by the mechanical balance.

Parallel shear induces (div $\mathbf{E}_\perp$)

$$(\nabla \times \mathbf{v})_{//} \propto (\nabla \cdot \mathbf{E}_\perp)\hat{e}_B$$

$$\mathbf{v}_\perp = \frac{\mathbf{E} \times \hat{e}_B}{B_0}$$



perpendicular shear produces (rot $\mathbf{E}_\perp$)

$$\frac{\partial}{\partial s}\mathbf{v}_\perp \propto (\nabla \times \mathbf{v})_\perp \propto (\nabla \times \mathbf{E}_\perp)_\perp$$

Always inductive!!

Note: $\nabla \cdot (\nabla \times \mathbf{v}) = 0 \longleftrightarrow \nabla_{//} \cdot (\nabla \times \mathbf{v})_{//} = -\nabla_\perp \cdot (\nabla \times \mathbf{v})_\perp$

vorticity conserves as like a current

$$\nabla \cdot \mathbf{j} = 0 \longleftrightarrow \nabla_{//} \cdot \mathbf{j}_{//} = -\nabla_\perp \cdot \mathbf{j}_\perp$$

Shear Alfven wave

$$\mathbf{j}_\perp = B_0 \Sigma_A (\nabla \times \mathbf{v})_\perp$$

$$\nabla \cdot \mathbf{j}_\perp = 0 \longleftrightarrow \nabla \cdot (\nabla \times \mathbf{v}) = 0$$

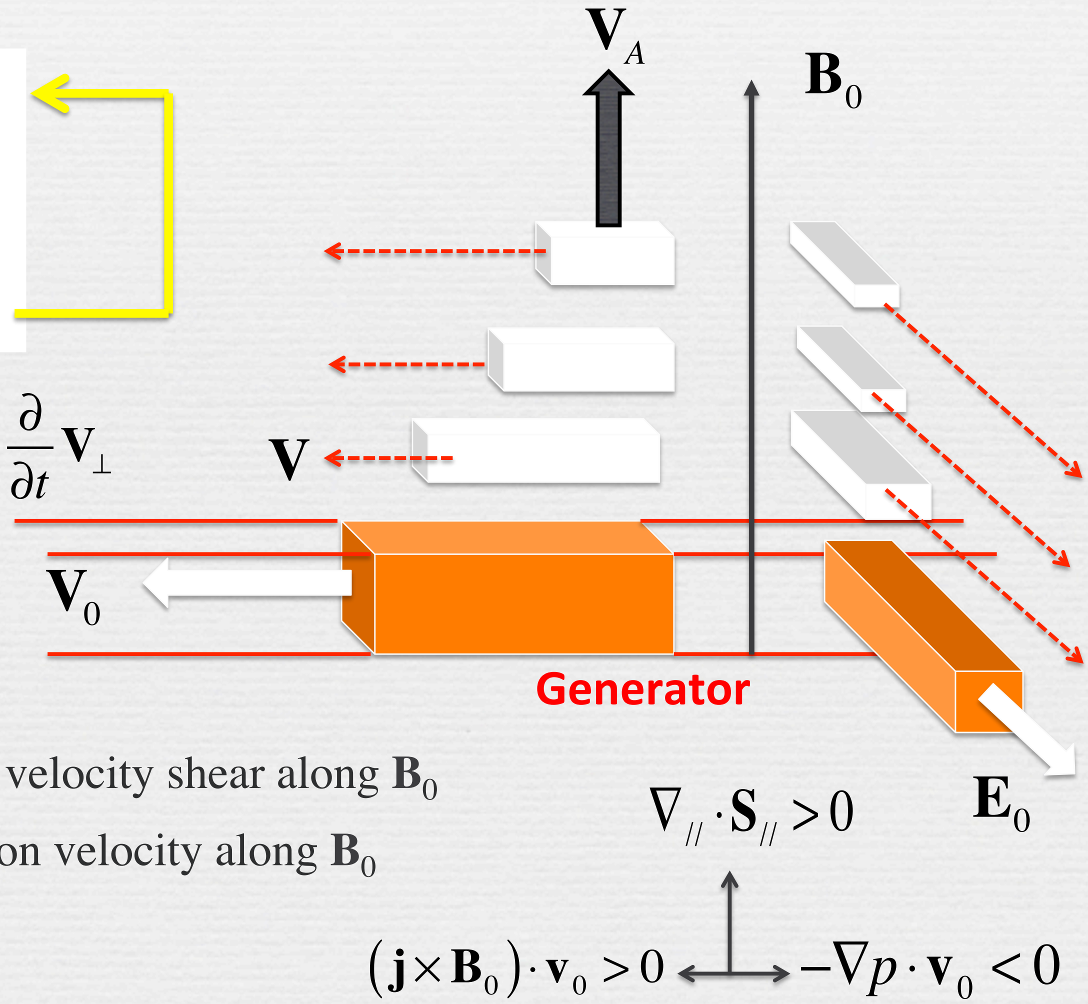
$$\mathbf{j}_{//} = B_0 \Sigma_A (\nabla \times \mathbf{v})_{//}$$

Induction process: generation and propagation shear Alfven wave

(1) Generator : drives enhanced convection velocity by balance of acceleration by pressure gradient force and deceleration by the magnetic tension force

(2) velocity shear along corresponds to rot(E)
→ development of magnetic field
→ development of magnetic tension
→ acceleration of plasma flow at wave front

$$\frac{\partial}{\partial s}\mathbf{V}_{\perp} \propto \left(\nabla\times\mathbf{E}\right)_{\perp} \propto \frac{\partial}{\partial t}\mathbf{b}_{\perp} \rightarrow \mathbf{j}\times\mathbf{B}_0 \rightarrow \rho\frac{\partial}{\partial t}\mathbf{V}_{\perp}$$



- role of Alfven wave → disappearance of velocity shear along $\mathbf{B}_0$
- Generator acts until difference of convection velocity along $\mathbf{B}_0$ is disappeared

Wave reflection by plasma inhomogeneity

shear Alfven wave incident on higher density plasma region

↓

Magnetic tension with incident wave don't accelerate in the level of generator velocity
(braking of plasma flow)

↓

Generation of plasma flow in the opposite direction

↓

generation of reflected shear Alfven wave

↓

Incident +reflection → deceleration of convection velocity

Propagation of shear Alfven wave inside the ionosphere

・Magnetic tension produced by the wave front current accelerates the plasma in the ionosphere and accelerated plasma is gradually braking by the plasma (ion) collision to the neutral.

$$\rho \frac{\partial}{\partial t}\mathbf{v} = \mathbf{j}\times\mathbf{B}_0 - \rho\nu_{in}\mathbf{v}$$

・This means perpendicular current density is not only composed of the polarization current but also by the braking current, of which directions are opposite each other.

$$\mathbf{j}_\perp = -\frac{\rho}{B_0^2}\frac{\partial}{\partial t}\mathbf{v}\times\mathbf{B}_0 - \frac{\rho\nu_{in}}{B_0^2}\mathbf{v}\times\mathbf{B}_0$$

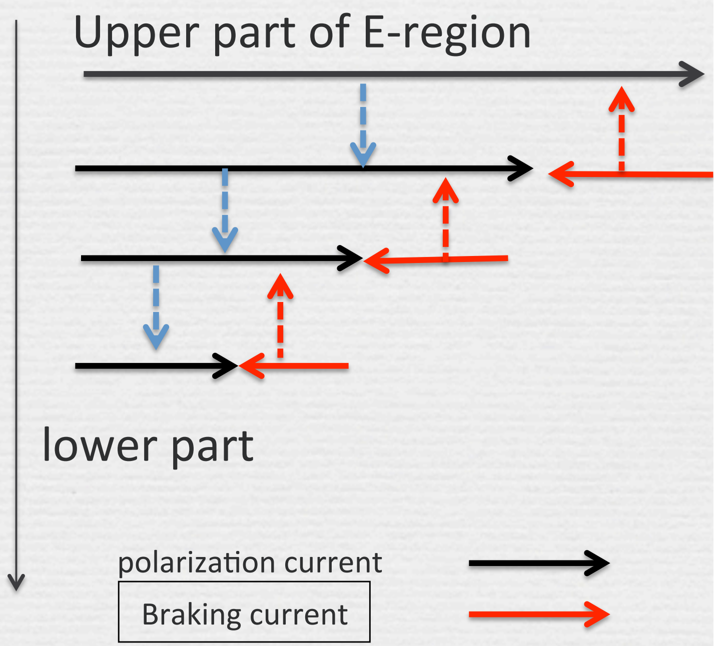
polarization current | Braking current

・Braking current radiates the shear Alfven wave by **induction loop**

・After wave front passage, accelerated plasma at the wave front is slower than that of back ward plasma velocity at the magnetospheric side

・Resultant total perpendicular current is gradually decreasing during wave passage in the ionosphere

・M-I coupling process via shear Alfven wave would terminated when the perpendicular current produced by the wave front current cannot accelerate the ambient plasma.



plasma – neutral interaction

$$\mathbf{j}\times\mathbf{B}\cong\rho\nu_{in}\left(\mathbf{V}-\mathbf{u}_n\right)$$

Equivalence of electromagnetic load and mechanical load

In the neutral frame

$$\left(\mathbf{j}\times\mathbf{B}\right)\cdot\mathbf{V}''\cong\rho\nu_{in}\left|\mathbf{V}''\right|^2 \qquad \mathbf{V}'\cong\mathbf{V}-\mathbf{u}_n$$

Work done by Ampere force = heating by plasma

In the rest (plasma) frame

Work done by Ampere force = work done by momentum exchange by collision

$$\left(\mathbf{j}\times\mathbf{B}\right)\cdot\mathbf{v}\cong\rho\nu_{in}\left(\mathbf{V}-\mathbf{u}_n\right)\cdot\mathbf{v}=\mathbf{j}\cdot\mathbf{E}$$

$$\mathbf{j}\cdot\mathbf{E}>0 \quad \mathit{for} \quad \mathbf{v}>\mathbf{u}_n$$

Atmospheric load

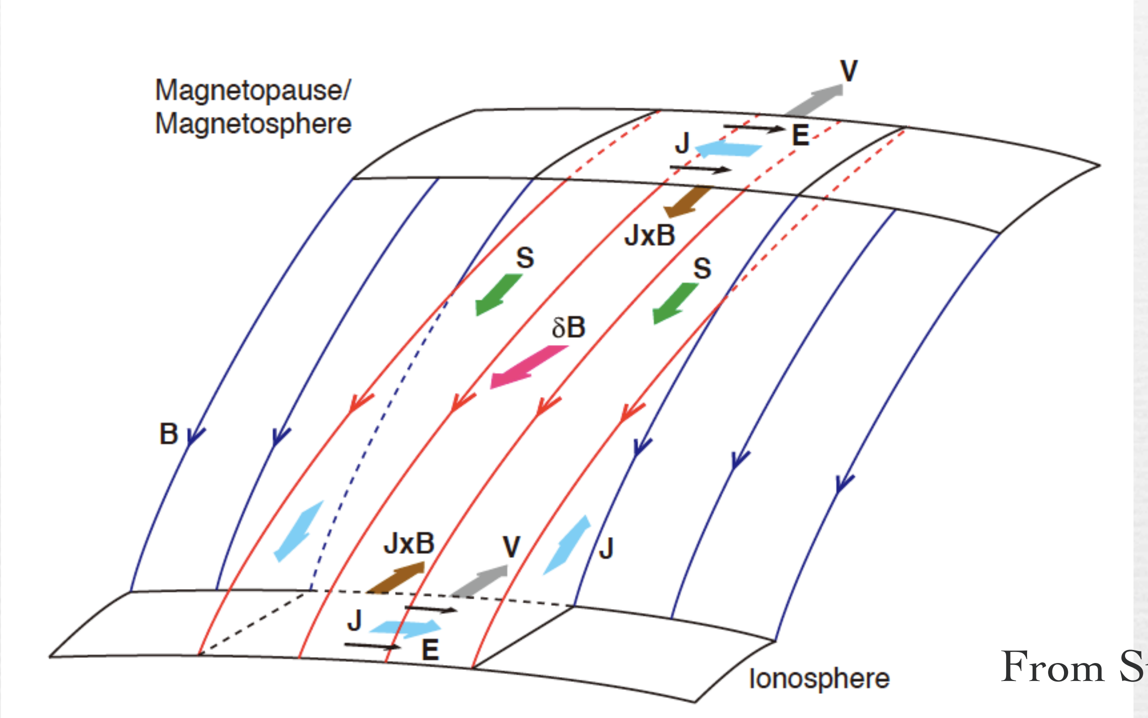
Electromagnetic energy is converted into mechanical energy

$$\mathbf{j}\cdot\mathbf{E}<0 \quad \mathit{for} \quad \mathbf{v}<\mathbf{u}_n$$

Atmospheric dynamo

Electromagnetic energy is generated from mechanical energy

Steady M-I coupling



From Strangeway [2001]

Enhanced flow at high altitude generator

Bending of field line (dB).

JxB force in generator . J opposite to –VxB (J.E<0)

FAC at the edges of shear region

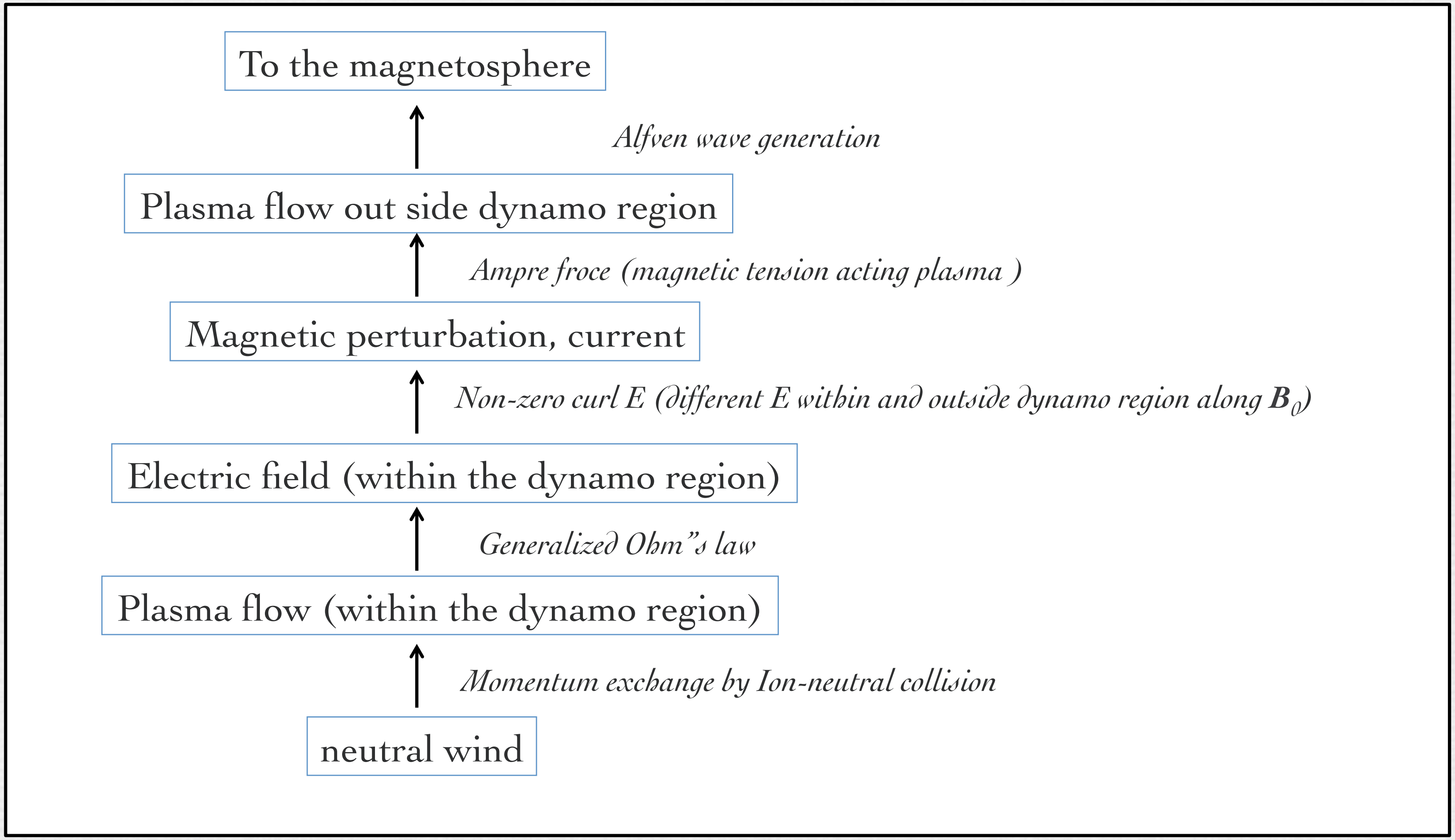
Current closure requires J at in ionosphere

Current loop gives dB required by field bending

Poynting flux (S=ExdB) into the ionosphere

Friction heating in ionosphere

Atmospheric dynamo



Transition of Ionospheric Ohm's law

$$\mathbf{E}+\mathbf{v}\times\mathbf{B}-\frac{\mathbf{j}\times\mathbf{B}}{en}=0$$ (electron frozen-in)

At the lower ionospheric region (E-region and below), mechanical balance between jxB force and collisional damping becomes dominant

$$\mathbf{j}\times\mathbf{B}_0 \simeq \rho\nu_{in}\mathbf{v}$$

convection electric field can be replaced by resistive electric field by ions collision to neutrals

$$-\mathbf{v}\times\mathbf{B}\simeq\frac{(\mathbf{j}\times\mathbf{B}_0)\times\mathbf{B}_0}{\rho\nu_{in}}=\frac{B_0^2}{\rho\nu_{in}}\mathbf{j}_\perp=\left(\frac{\Omega_i}{\nu_{in}}\right)\left(\frac{B_0}{en}\right)\mathbf{j}_\perp \qquad \Omega_i<\nu_{in}$$

Hall electric field becomes important at lower ionosphere satisfies

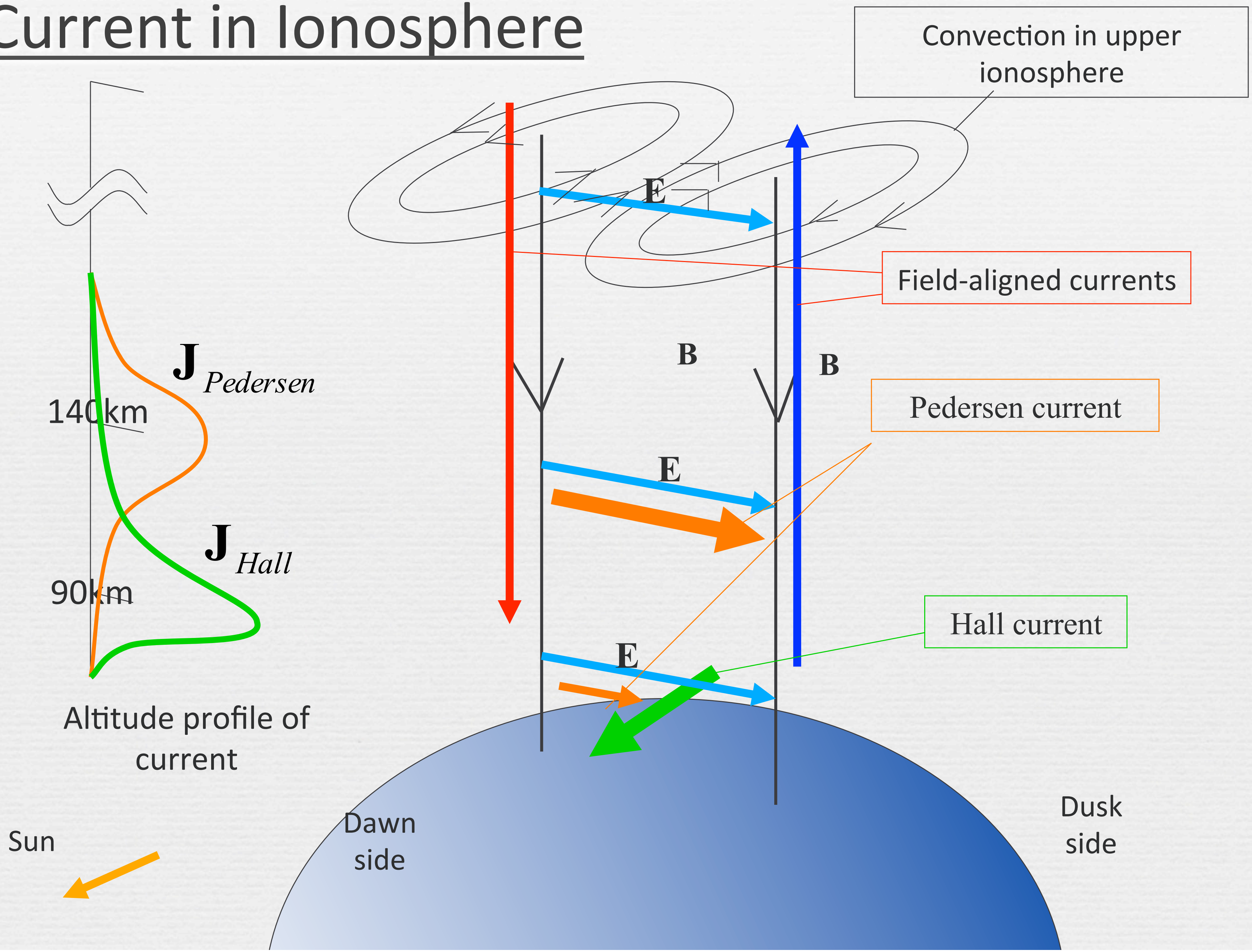
$$\mathbf{E}_\perp=\left(\frac{B_0}{en}\right)\left(\frac{\Omega_i}{\nu_{in}}\mathbf{j}_\perp+\mathbf{j}_\perp\times\hat{b}\right)$$

Resistive electric field (E_R)

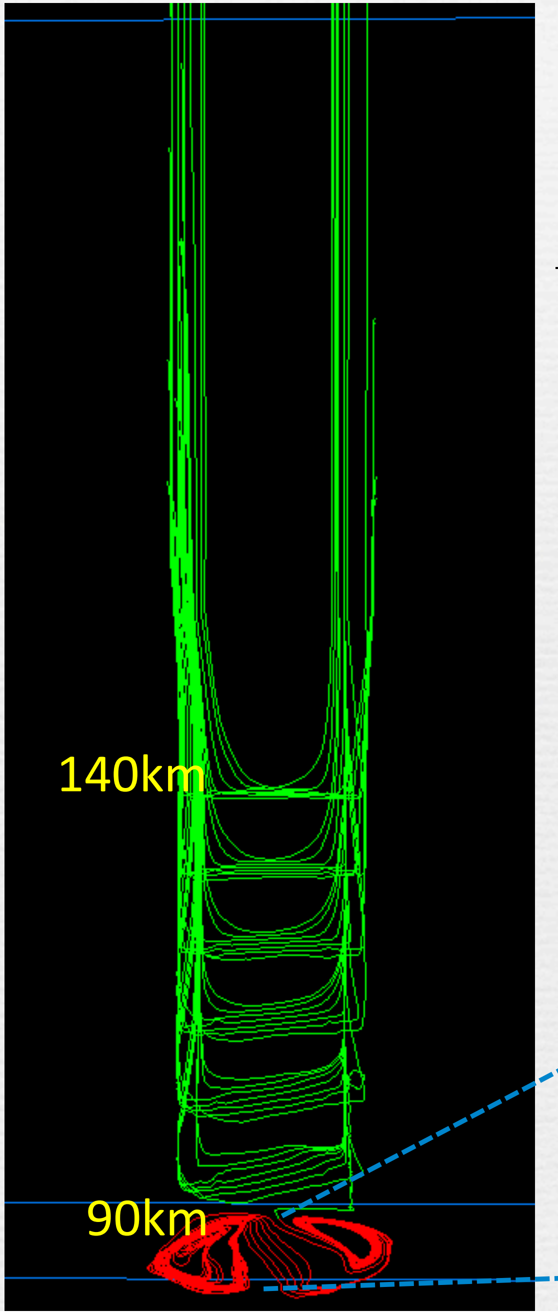
Hall electric field (E_H)

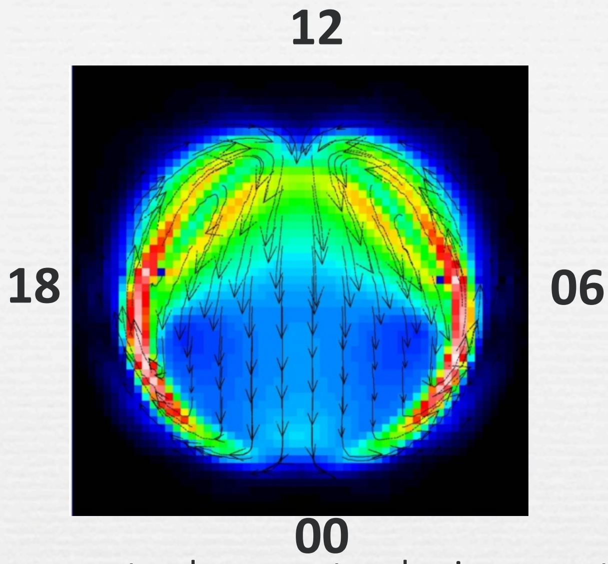
rot$(E_H)_\perp$ develops the horizontally closing current!!

Current in Ionosphere



3D current closure produced by Hall MHD simulation (with horizontally homogeneous plasma distribution)



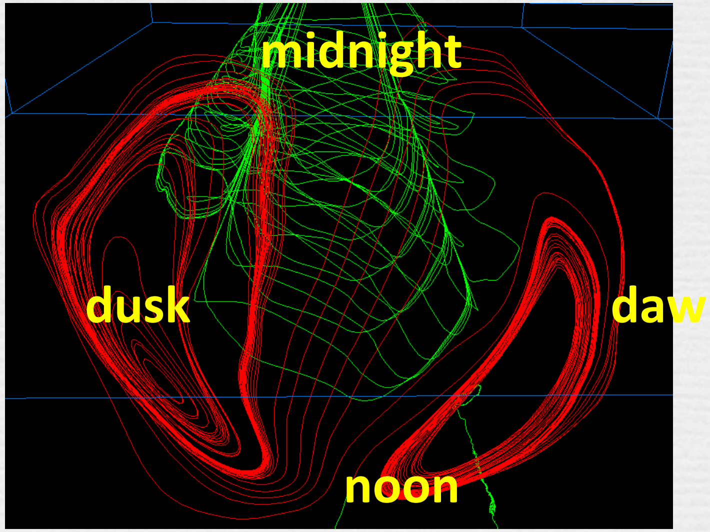


$$\frac{\Omega_i}{\nu_{in}} >> 1 \quad \mathbf{E}_\perp \simeq -\mathbf{V} \times \mathbf{B}_0$$

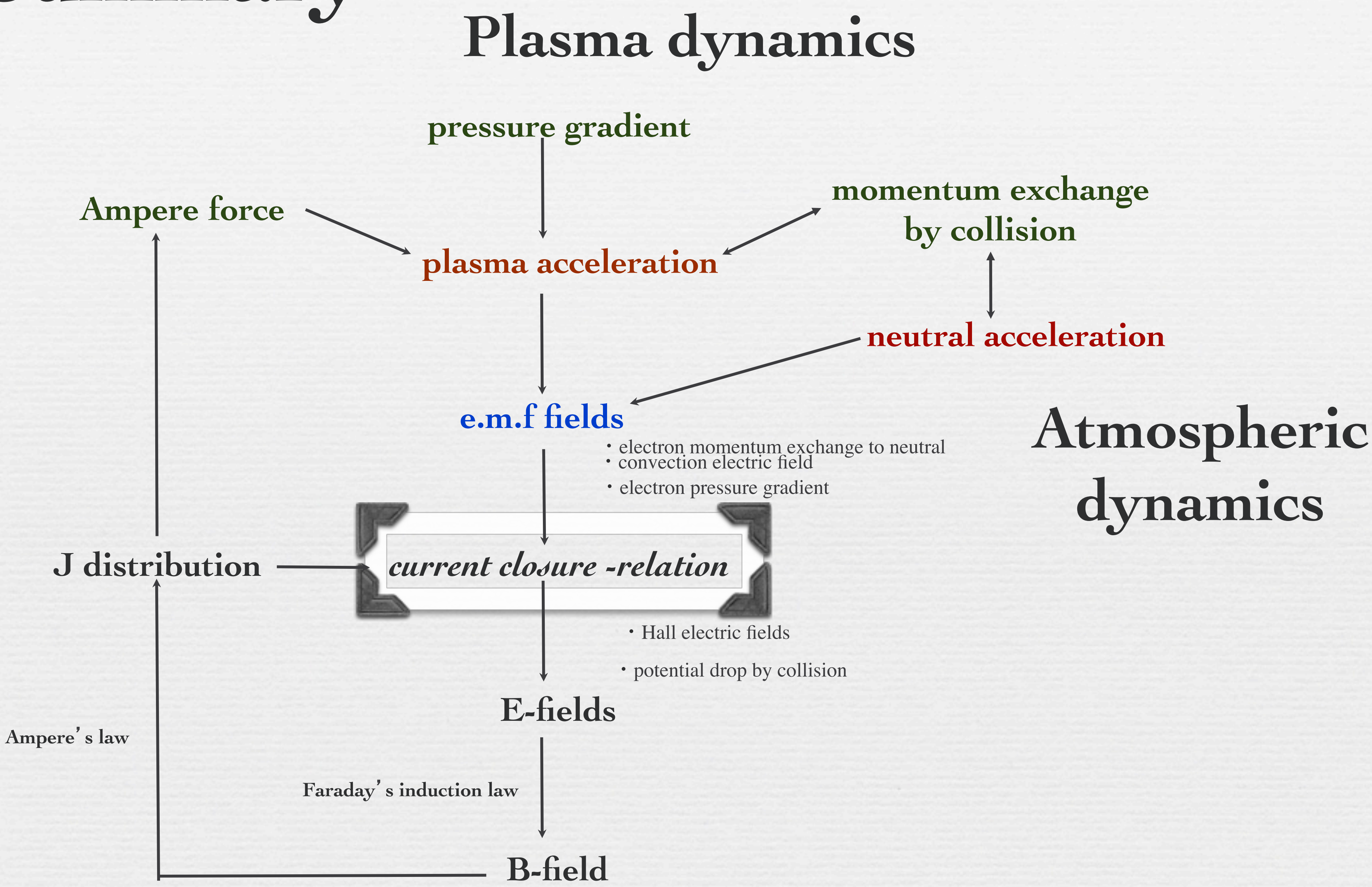
- Upper boundary condition uses steady magnetospheric convection obtained by global MHD simulation
- Field aligned currents are closed at altitudes of 90km～140km.
- Loop current could be identified in bottom regions of model.

$$\frac{\Omega_i}{\nu_{in}} \sim 1 \quad \mathbf{E}_\perp \simeq \left(\frac{B_0}{en}\right)\left(\frac{\Omega_i}{\nu_{in}} \mathbf{j}_\perp\right)$$

$$\frac{\Omega_i}{\nu_{in}} << 1 \quad \mathbf{E}_\perp \simeq \left(\frac{B_0}{en}\right)\left(\mathbf{j}_\perp \times \hat{e}_B\right)$$



Summary


Plasma dynamics
pressure gradient
Ampere force
momentum exchange
by collision
plasma acceleration
neutral acceleration
e.m.f fields
・electron momentum exchange to neutral
・convection electric field
・electron pressure gradient
Atmospheric
dynamics
J distribution
current closure -relation
・Hall electric fields
・potential drop by collision
E-fields
Ampere's law
Faraday's induction law
B-field